Chapter 13

Fundamental Equilibrium Concepts

Figure 13.1 Movement of carbon dioxide through tissues and blood cells involves several equilibrium reactions.

Chapter Outline

13.1 Chemical Equilibria
13.2 Equilibrium Constants
13.3 Shifting Equilibria: Le Châtelier’s Principle
13.4 Equilibrium Calculations

Introduction

Imagine a beach populated with sunbathers and swimmers. As those basking in the sun get too hot and want to cool off, they head into the surf to swim. As the swimmers tire, they head to the beach to rest. If these two rates of transfer (sunbathers entering the water, swimmers leaving the water) are equal, the number of sunbathers and swimmers would be constant, or at equilibrium, although the identities of the people are constantly changing from sunbather to swimmer and back. An analogous situation occurs in chemical reactions. Reactions can occur in both directions simultaneously (reactants to products and products to reactants) and eventually reach a state of balance.

These balanced two-way reactions occur all around and even in us. For example, they occur in our blood, where the reaction between carbon dioxide and water forms carbonic acid (\(\text{HCO}_3^-\)) (Figure 13.1). Human physiology is adapted to the amount of ionized products produced by this reaction (\(\text{HCO}_3^-\) and \(\text{H}^+\)). In this chapter, you will learn how to predict the position of the balance and the yield of a product of a reaction under specific conditions, how to change a reaction's conditions to increase or reduce yield, and how to evaluate an equilibrium system's reaction to disturbances.
13.1 Chemical Equilibria

By the end of this section, you will be able to:

- Describe the nature of equilibrium systems
- Explain the dynamic nature of a chemical equilibrium

A chemical reaction is usually written in a way that suggests it proceeds in one direction, the direction in which we read, but all chemical reactions are reversible, and both the forward and reverse reaction occur to one degree or another depending on conditions. In a chemical equilibrium, the forward and reverse reactions occur at equal rates, and the concentrations of products and reactants remain constant. If we run a reaction in a closed system so that the products cannot escape, we often find the reaction does not give a 100% yield of products. Instead, some reactants remain after the concentrations stop changing. At this point, when there is no further change in concentrations of reactants and products, we say the reaction is at equilibrium. A mixture of reactants and products is found at equilibrium.

For example, when we place a sample of dinitrogen tetroxide (N$_2$O$_4$, a colorless gas) in a glass tube, it forms nitrogen dioxide (NO$_2$, a brown gas) by the reaction

$$\text{N}_2\text{O}_4(g) \rightleftharpoons 2\text{NO}_2(g)$$

The color becomes darker as N$_2$O$_4$ is converted to NO$_2$. When the system reaches equilibrium, both N$_2$O$_4$ and NO$_2$ are present (Figure 13.2).
Figure 13.2 A mixture of NO\(_2\) and N\(_2\)O\(_4\) moves toward equilibrium. Colorless N\(_2\)O\(_4\) reacts to form brown NO\(_2\). As the reaction proceeds toward equilibrium, the color of the mixture darkens due to the increasing concentration of NO\(_2\).

The formation of NO\(_2\) from N\(_2\)O\(_4\) is a \textbf{reversible reaction}, which is identified by the equilibrium arrow (\(\rightleftharpoons\)). All reactions are reversible, but many reactions, for all practical purposes, proceed in one direction until the reactants are exhausted and will reverse only under certain conditions. Such reactions are often depicted with a one-way arrow from reactants to products. Many other reactions, such as the formation of NO\(_2\) from N\(_2\)O\(_4\), are reversible under more easily obtainable conditions and, therefore, are named as such. In a reversible reaction, the reactants can combine to form products and the products can react to form the reactants. Thus, not only can N\(_2\)O\(_4\) decompose to form NO\(_2\), but the NO\(_2\) produced can react to form N\(_2\)O\(_4\). As soon as the forward reaction produces any NO\(_2\), the reverse reaction begins and NO\(_2\) starts to react to form N\(_2\)O\(_4\). At equilibrium, the concentrations of N\(_2\)O\(_4\) and NO\(_2\) no longer change because the rate of formation of NO\(_2\) is exactly equal to the rate of consumption of NO\(_2\), and the rate of formation of N\(_2\)O\(_4\) is exactly equal to the rate of consumption of N\(_2\)O\(_4\). \textit{Chemical equilibrium is a dynamic process}: As with the swimmers and the sunbathers, the numbers of each remain constant, yet there is a flux back and forth between them (Figure 13.3).
In a chemical equilibrium, the forward and reverse reactions do not stop, rather they continue to occur at the same rate, leading to constant concentrations of the reactants and the products. Plots showing how the reaction rates and concentrations change with respect to time are shown in Figure 13.2.

We can detect a state of equilibrium because the concentrations of reactants and products do not appear to change. However, it is important that we verify that the absence of change is due to equilibrium and not to a reaction rate that is so slow that changes in concentration are difficult to detect.

We use a double arrow when writing an equation for a reversible reaction. Such a reaction may or may not be at equilibrium. For example, Figure 13.2 shows the reaction:

$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$

When we wish to speak about one particular component of a reversible reaction, we use a single arrow. For example, in the equilibrium shown in Figure 13.2, the rate of the forward reaction

$$2NO_2(g) \rightarrow N_2O_4(g)$$

is equal to the rate of the backward reaction

$$N_2O_4(g) \rightarrow 2NO_2(g)$$

**Chemistry in Everyday Life**

**Equilibrium and Soft Drinks**

The connection between chemistry and carbonated soft drinks goes back to 1767, when Joseph Priestley (1733–1804; mostly known today for his role in the discovery and identification of oxygen) discovered a method of infusing water with carbon dioxide to make carbonated water. In 1772, Priestly published a paper entitled "Impregnating Water with Fixed Air." The paper describes dripping oil of vitriol (today we call this sulfuric acid, but what a great way to describe sulfuric acid: "oil of vitriol" literally means "liquid nastiness") onto chalk (calcium carbonate). The resulting CO$_2$ falls into the container of water beneath the vessel in which the initial reaction takes place; agitation helps the gaseous CO$_2$ mix into the liquid water.

$$H_2SO_4(l) + CaCO_3(s) \rightarrow CO_2(g) + H_2O(l) + CaSO_4(aq)$$
Carbon dioxide is slightly soluble in water. There is an equilibrium reaction that occurs as the carbon dioxide reacts with the water to form carbonic acid ($H_2CO_3$). Since carbonic acid is a weak acid, it can dissociate into protons ($H^+$) and hydrogen carbonate ions ($HCO_3^-$).

$$CO_2(aq) + H_2O(l) \rightleftharpoons H_2CO_3(aq) \rightleftharpoons HCO_3^-(aq) + H^+(aq)$$

Today, CO$_2$ can be pressurized into soft drinks, establishing the equilibrium shown above. Once you open the beverage container, however, a cascade of equilibrium shifts occurs. First, the CO$_2$ gas in the air space on top of the bottle escapes, causing the equilibrium between gas-phase CO$_2$ and dissolved or aqueous CO$_2$ to shift, lowering the concentration of CO$_2$ in the soft drink. Less CO$_2$ dissolved in the liquid leads to carbonic acid decomposing to dissolved CO$_2$ and H$_2$O. The lowered carbonic acid concentration causes a shift of the final equilibrium. As long as the soft drink is in an open container, the CO$_2$ bubbles up out of the beverage, releasing the gas into the air (Figure 13.4). With the lid off the bottle, the CO$_2$ reactions are no longer at equilibrium and will continue until no more of the reactants remain. This results in a soft drink with a much lowered CO$_2$ concentration, often referred to as “flat.”

![Figure 13.4](https://example.com/figure13_4.jpg)  
When a soft drink is opened, several equilibrium shifts occur. (credit: modification of work by "D Coetzee"/Flickr)

Let us consider the evaporation of bromine as a second example of a system at equilibrium.

$$Br_2(l) \rightleftharpoons Br_2(g)$$

An equilibrium can be established for a physical change—like this liquid to gas transition—as well as for a chemical reaction. Figure 13.5 shows a sample of liquid bromine at equilibrium with bromine vapor in a closed container. When we pour liquid bromine into an empty bottle in which there is no bromine vapor, some liquid evaporates, the amount of liquid decreases, and the amount of vapor increases. If we cap the bottle so no vapor escapes, the amount of liquid and vapor will eventually stop changing and an equilibrium between the liquid and the vapor will be established. If the bottle were not capped, the bromine vapor would escape and no equilibrium would be reached.
13.2 Equilibrium Constants

By the end of this section, you will be able to:

- Derive reaction quotients from chemical equations representing homogeneous and heterogeneous reactions
- Calculate values of reaction quotients and equilibrium constants, using concentrations and pressures
- Relate the magnitude of an equilibrium constant to properties of the chemical system

Now that we have a symbol (⇌) to designate reversible reactions, we will need a way to express mathematically how the amounts of reactants and products affect the equilibrium of the system. A general equation for a reversible reaction may be written as follows:

\[ mA + nB \rightleftharpoons xC + yD \]

We can write the reaction quotient \( Q \) for this equation. When evaluated using concentrations, it is called \( Q_c \). We use brackets to indicate molar concentrations of reactants and products.

\[ Q_c = \frac{[C]^x[D]^y}{[A]^m[B]^n} \]

The reaction quotient is equal to the molar concentrations of the products of the chemical equation (multiplied together) over the reactants (also multiplied together), with each concentration raised to the power of the coefficient of that substance in the balanced chemical equation. For example, the reaction quotient for the reversible reaction \( 2\text{NO}_2(g) \rightleftharpoons \text{N}_2\text{O}_4(g) \) is given by this expression:
\[ Q_c = \frac{[\text{N}_2\text{O}_4]}{[\text{NO}_2]^2} \]

**Example 13.1**

**Writing Reaction Quotient Expressions**

Write the expression for the reaction quotient for each of the following reactions:

(a) \(3\text{O}_2(g) \rightleftharpoons 2\text{O}_3(g)\)
(b) \(\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)\)
(c) \(4\text{NH}_3(g) + 7\text{O}_2(g) \rightleftharpoons 4\text{NO}_2(g) + 6\text{H}_2\text{O}(g)\)

**Solution**

(a) \(Q_c = \frac{[\text{O}_3]^2}{[\text{O}_2]^3}\)
(b) \(Q_c = \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3}\)
(c) \(Q_c = \frac{[\text{NO}_2]^4[\text{H}_2\text{O}]^6}{[\text{NH}_3]^4[\text{O}_2]^7}\)

**Check Your Learning**

Write the expression for the reaction quotient for each of the following reactions:

(a) \(2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)\)
(b) \(\text{C}_4\text{H}_8(g) \rightleftharpoons 2\text{C}_2\text{H}_4(g)\)
(c) \(2\text{C}_4\text{H}_{10}(g) + 13\text{O}_2(g) \rightleftharpoons 8\text{CO}_2(g) + 10\text{H}_2\text{O}(g)\)

Answer: (a) \(Q_c = \frac{[\text{SO}_3]^2}{[\text{SO}_2]^2[\text{O}_2]}\)
(b) \(Q_c = \frac{[\text{C}_2\text{H}_4]^2}{[\text{C}_4\text{H}_8]}\)
(c) \(Q_c = \frac{[\text{CO}_2]^8[\text{H}_2\text{O}]^{10}}{[\text{C}_4\text{H}_{10}]^2[\text{O}_2]^{13}}\)

The numeric value of \(Q_c\) for a given reaction varies; it depends on the concentrations of products and reactants present at the time when \(Q_c\) is determined. When pure reactants are mixed, \(Q_c\) is initially zero because there are no products present at that point. As the reaction proceeds, the value of \(Q_c\) increases as the concentrations of the products increase and the concentrations of the reactants simultaneously decrease (Figure 13.6). When the reaction reaches equilibrium, the value of the reaction quotient no longer changes because the concentrations no longer change.
Figure 13.6 (a) The change in the concentrations of reactants and products is depicted as the reaction $2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)$ approaches equilibrium. (b) The change in concentrations of reactants and products is depicted as the reaction $2\text{SO}_3(g) \rightleftharpoons 2\text{SO}_2(g) + \text{O}_2(g)$ approaches equilibrium. (c) The graph shows the change in the value of the reaction quotient as the reaction approaches equilibrium.

When a mixture of reactants and products of a reaction reaches equilibrium at a given temperature, its reaction quotient always has the same value. This value is called the equilibrium constant ($K$) of the reaction at that temperature. As for the reaction quotient, when evaluated in terms of concentrations, it is noted as $K_c$.

That a reaction quotient always assumes the same value at equilibrium can be expressed as:

$$Q_c \text{ at equilibrium} = K_c = \frac{[C]^y[D]^z...}{[A]^m[B]^n...}$$

This equation is a mathematical statement of the law of mass action: When a reaction has attained equilibrium at a given temperature, the reaction quotient for the reaction always has the same value.
Example 13.2

Evaluating a Reaction Quotient

Gaseous nitrogen dioxide forms dinitrogen tetroxide according to this equation:

\[ 2\text{NO}_2(g) \rightleftharpoons \text{N}_2\text{O}_4(g) \]

When 0.10 mol NO\(_2\) is added to a 1.0-L flask at 25 °C, the concentration changes so that at equilibrium, [NO\(_2\)] = 0.016 M and [N\(_2\)O\(_4\)] = 0.042 M.

(a) What is the value of the reaction quotient before any reaction occurs?

(b) What is the value of the equilibrium constant for the reaction?

Solution

(a) Before any product is formed, \([\text{NO}_2] = \frac{0.10 \text{ mol}}{1.0 \text{ L}} = 0.10 \text{ M}, \text{ and } [\text{N}_2\text{O}_4] = 0 \text{ M}. \) Thus,

\[ Q_c = \frac{[\text{N}_2\text{O}_4]}{[\text{NO}_2]^2} = \frac{0}{0.10^2} = 0 \]

(b) At equilibrium, the value of the equilibrium constant is equal to the value of the reaction quotient. At equilibrium, \( K_c = Q_c = \frac{[\text{N}_2\text{O}_4]}{[\text{NO}_2]^2} = \frac{0.042}{0.016^2} = 1.6 \times 10^2 \). The equilibrium constant is \( 1.6 \times 10^2 \).

Note that dimensional analysis would suggest the unit for this \( K_c \) value should be \( M^{-1} \). However, it is common practice to omit units for \( K_c \) values computed as described here, since it is the magnitude of an equilibrium constant that relays useful information. As will be discussed later in this module, the rigorous approach to computing equilibrium constants uses dimensionless quantities derived from concentrations instead of actual concentrations, and so \( K_c \) values are truly unitless.

Check Your Learning

For the reaction \( 2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g) \), the concentrations at equilibrium are \([\text{SO}_2] = 0.90 \text{ M}, [\text{O}_2] = 0.35 \text{ M}, \text{ and } [\text{SO}_3] = 1.1 \text{ M}. \) What is the value of the equilibrium constant, \( K_c \)?

Answer: \( K_c = 4.3 \)

The magnitude of an equilibrium constant is a measure of the yield of a reaction when it reaches equilibrium. A large value for \( K_c \) indicates that equilibrium is attained only after the reactants have been largely converted into products. A small value of \( K_c \)—much less than 1—indicates that equilibrium is attained when only a small proportion of the reactants have been converted into products.

Once a value of \( K_c \) is known for a reaction, it can be used to predict directional shifts when compared to the value of \( Q_c \). A system that is not at equilibrium will proceed in the direction that establishes equilibrium. The data in Figure 13.7 illustrate this. When heated to a consistent temperature, 800 °C, different starting mixtures of CO, H\(_2\)O, CO\(_2\), and H\(_2\) react to reach compositions adhering to the same equilibrium (the value of \( Q_c \) changes until it equals the value of \( K_c \)). This value is 0.640, the equilibrium constant for the reaction under these conditions.

\[ \text{CO}(g) + \text{H}_2\text{O}(g) \rightleftharpoons \text{CO}_2(g) + \text{H}_2(g) \quad K_c = 0.640 \quad T = 800 ^\circ \text{C} \]

It is important to recognize that an equilibrium can be established starting either from reactants or from products, or from a mixture of both. For example, equilibrium was established from Mixture 2 in Figure 13.7 when the products of the reaction were heated in a closed container. In fact, one technique used to determine whether a reaction is truly at equilibrium is to approach equilibrium starting with reactants in one experiment and starting with products in another. If the same value of the reaction quotient is observed when the concentrations stop changing in both experiments, then we may be certain that the system has reached equilibrium.
Figure 13.7  Concentrations of three mixtures are shown before and after reaching equilibrium at 800 °C for the so-called water gas shift reaction: \( \text{CO}(g) + \text{H}_2\text{O}(g) \rightleftharpoons \text{CO}_2(g) + \text{H}_2(g) \).

Example 13.3

Predicting the Direction of Reaction

Given here are the starting concentrations of reactants and products for three experiments involving this reaction:

\[
\text{CO}(g) + \text{H}_2\text{O}(g) \rightleftharpoons \text{CO}_2(g) + \text{H}_2(g)
\]

\[K_c = 0.64\]

Determine in which direction the reaction proceeds as it goes to equilibrium in each of the three experiments shown.

<table>
<thead>
<tr>
<th>Reactants/Products</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
<th>Experiment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>([\text{CO}]_i)</td>
<td>0.0203 (M)</td>
<td>0.011 (M)</td>
<td>0.0094 (M)</td>
</tr>
<tr>
<td>([\text{H}_2\text{O}]_i)</td>
<td>0.0203 (M)</td>
<td>0.0011 (M)</td>
<td>0.0025 (M)</td>
</tr>
<tr>
<td>([\text{CO}_2]_i)</td>
<td>0.0040 (M)</td>
<td>0.037 (M)</td>
<td>0.0015 (M)</td>
</tr>
<tr>
<td>([\text{H}_2]_i)</td>
<td>0.0040 (M)</td>
<td>0.046 (M)</td>
<td>0.0076 (M)</td>
</tr>
</tbody>
</table>

Solution

Experiment 1:

\[
Q_c = \frac{[\text{CO}_2][\text{H}_2]}{[\text{CO}][\text{H}_2\text{O}]} = \frac{(0.0040)(0.0040)}{(0.0203)(0.0203)} = 0.039.
\]

\(Q_c < K_c\) (0.039 < 0.64)

The reaction will shift to the right.

Experiment 2:

\[
Q_c = \frac{[\text{CO}_2][\text{H}_2]}{[\text{CO}][\text{H}_2\text{O}]} = \frac{(0.037)(0.046)}{(0.011)(0.0011)} = 1.4 \times 10^2
\]

\(Q_c > K_c\) (140 > 0.64)
The reaction will shift to the left.

Experiment 3:

\[ Q_c = \frac{[CO_2][H_2]}{[CO][H_2O]} = \frac{(0.0015)(0.0076)}{(0.0094)(0.0025)} = 0.48 \]

\[ Q_c < K_c \quad (0.48 < 0.64) \]

The reaction will shift to the right.

**Check Your Learning**

Calculate the reaction quotient and determine the direction in which each of the following reactions will proceed to reach equilibrium.

(a) A 1.00-L flask containing 0.0500 mol of NO(g), 0.0155 mol of Cl_2(g), and 0.500 mol of NOCl:

\[
2\text{NO}(g) + \text{Cl}_2(g) \rightleftharpoons 2\text{NOCl}(g) \\
K_c = 4.6 \times 10^4
\]

(b) A 5.0-L flask containing 17 g of NH_3, 14 g of N_2, and 12 g of H_2:

\[
\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g) \\
K_c = 0.060
\]

(c) A 2.00-L flask containing 230 g of SO_3(g):

\[
2\text{SO}_3(g) \rightleftharpoons 2\text{SO}_2(g) + \text{O}_2(g) \\
K_c = 0.230
\]

**Answer:** (a) \( Q_c = 6.45 \times 10^3 \), shifts right. (b) \( Q_c = 0.12 \), shifts left. (c) \( Q_c = 0 \), shifts right

In Example 13.2, it was mentioned that the common practice is to omit units when evaluating reaction quotients and equilibrium constants. It should be pointed out that using concentrations in these computations is a convenient but simplified approach that sometimes leads to results that seemingly conflict with the law of mass action. For example, equilibria involving aqueous ions often exhibit equilibrium constants that vary quite significantly (are not constant) at high solution concentrations. This may be avoided by computing \( K_c \) values using the activities of the reactants and products in the equilibrium system instead of their concentrations. The activity of a substance is a measure of its effective concentration under specified conditions. While a detailed discussion of this important quantity is beyond the scope of an introductory text, it is necessary to be aware of a few important aspects:

- Activities are dimensionless (unitless) quantities and are in essence “adjusted” concentrations.
- For relatively dilute solutions, a substance’s activity and its molar concentration are roughly equal.
- Activities for pure condensed phases (solids and liquids) are equal to 1.

As a consequence of this last consideration, \( Q_c \) and \( K_c \) expressions do not contain terms for solids or liquids (being numerically equal to 1, these terms have no effect on the expression’s value). Several examples of equilibria yielding such expressions will be encountered in this section.

**Homogeneous Equilibria**

A **homogeneous equilibrium** is one in which all of the reactants and products are present in a single solution (by definition, a homogeneous mixture). In this chapter, we will concentrate on the two most common types of homogeneous equilibria: those occurring in liquid-phase solutions and those involving exclusively gaseous species. Reactions between solutes in liquid solutions belong to one type of homogeneous equilibria. The chemical species involved can be molecules, ions, or a mixture of both. Several examples are provided here.

\[
\text{C}_2\text{H}_2(aq) + 2\text{Br}_2(aq) \rightleftharpoons \text{C}_2\text{H}_2\text{Br}_4(aq) \\
K_c = \frac{[\text{C}_2\text{H}_2\text{Br}_4]}{[\text{C}_2\text{H}_2][\text{Br}_2]^2}
\]

\[
\text{I}_2(aq) + \text{I}^-(aq) \rightleftharpoons \text{I}_3^-(aq) \\
K_c = \frac{[\text{I}_3^-]}{[\text{I}_2][\text{I}^-]}
\]
Hg$_2^{2+}$(aq) + NO$_3^-$ (aq) + 3H$_2$O$^+$ (aq) ⇌ 2Hg$^{2+}$(aq) + HNO$_2$(aq) + 4H$_2$O(l)

\[ K_c = \frac{[\text{Hg}_2^{2+}]^2[\text{HNO}_2]}{[\text{Hg}_2^{2+}]^2[\text{NO}_3^-][\text{H}_2\text{O}^+]^3} \]

HF(aq) + H$_2$O(l) ⇌ H$_3$O$^+$ (aq) + F$^-$ (aq)

\[ K_c = \frac{[\text{H}_3\text{O}^+][\text{F}^-]}{[\text{HF}]} \]

NH$_3$(aq) + H$_2$O(l) ⇌ NH$_4^+$ (aq) + OH$^-$ (aq)

\[ K_c = \frac{[\text{NH}_4^+][\text{OH}^-]}{[\text{NH}_3]} \]

In each of these examples, the equilibrium system is an aqueous solution, as denoted by the $aq$ annotations on the solute formulas. Since H$_2$O(l) is the solvent for these solutions, its concentration does not appear as a term in the $K_c$ expression, as discussed earlier, even though it may also appear as a reactant or product in the chemical equation.

Reactions in which all reactants and products are gases represent a second class of homogeneous equilibria. We use molar concentrations in the following examples, but we will see shortly that partial pressures of the gases may be used as well.

\[
\begin{align*}
\text{C}_2\text{H}_6(g) &\rightleftharpoons \text{C}_2\text{H}_4(g) + \text{H}_2(g) & K_c &= \frac{[\text{C}_2\text{H}_4][\text{H}_2]}{[\text{C}_2\text{H}_6]} \\
3\text{O}_2(g) &\rightarrow 2\text{O}_3(g) & K_c &= \frac{[\text{O}_3]^2}{[\text{O}_2]^3} \\
\text{N}_2(g) + 3\text{H}_2(g) &\rightarrow 2\text{NH}_3(g) & K_c &= \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3} \\
\text{C}_3\text{H}_8(g) + 5\text{O}_2(g) &\rightleftharpoons 3\text{CO}_2(g) + 4\text{H}_2\text{O}(g) & K_c &= \frac{[\text{CO}_2]^3[\text{H}_2\text{O}]^4}{[\text{C}_3\text{H}_8][\text{O}_2]^5} 
\end{align*}
\]

Note that the concentration of H$_2$O(g) has been included in the last example because water is not the solvent in this gas-phase reaction and its concentration (and activity) changes.

Whenever gases are involved in a reaction, the partial pressure of each gas can be used instead of its concentration in the equation for the reaction quotient because the partial pressure of a gas is directly proportional to its concentration at constant temperature. This relationship can be derived from the ideal gas equation, where $M$ is the molar concentration of gas, $\frac{n}{V}$.

\[
PV = nRT \\
P = \left(\frac{n}{V}\right)RT \\
= MRT
\]

Thus, at constant temperature, the pressure of a gas is directly proportional to its concentration.

Using the partial pressures of the gases, we can write the reaction quotient for the system C$_2$H$_6(g) \rightleftharpoons$ C$_2$H$_4(g) + H_2(g)$ by following the same guidelines for deriving concentration-based expressions:

\[ Q_P = \frac{P_{\text{C}_2\text{H}_4}P_{\text{H}_2}}{P_{\text{C}_2\text{H}_6}} \]

In this equation we use $Q_P$ to indicate a reaction quotient written with partial pressures: $P_{\text{C}_2\text{H}_6}$ is the partial pressure of C$_2$H$_6$; $P_{\text{H}_2}$ the partial pressure of H$_2$; and $P_{\text{C}_2\text{H}_4}$ the partial pressure of C$_2$H$_4$. At equilibrium:

\[ K_P = Q_P = \frac{P_{\text{C}_2\text{H}_4}P_{\text{H}_2}}{P_{\text{C}_2\text{H}_6}} \]

The subscript $P$ in the symbol $K_P$ designates an equilibrium constant derived using partial pressures instead of concentrations. The equilibrium constant, $K_P$, is still a constant, but its numeric value may differ from the equilibrium constant found for the same reaction by using concentrations.
Conversion between a value for \( K_c \), an equilibrium constant expressed in terms of concentrations, and a value for \( K_P \), an equilibrium constant expressed in terms of pressures, is straightforward (a \( K \) or \( Q \) without a subscript could be either concentration or pressure).

The equation relating \( K_c \) and \( K_P \) is derived as follows. For the gas-phase reaction \( mA + nB \rightleftharpoons xC + yD \):

\[
K_P = \frac{[P_C]^x [P_D]^y}{[P_A]^m [P_B]^n}
\]

\[
= \frac{([C] \times RT)^x ([D] \times RT)^y}{([A] \times RT)^m ([B] \times RT)^n}
\]

\[
= \frac{[C]^x[D]^y}{[A]^m[B]^n} \times \frac{(RT)^{x+y}}{(RT)^{m+n}}
\]

\[
= K_c (RT)^{(x+y) - (m+n)}
\]

\[
= K_c (RT)^{\Delta n}
\]

The relationship between \( K_c \) and \( K_P \) is

\[
K_P = K_c (RT)^{\Delta n}
\]

In this equation, \( \Delta n \) is the difference between the sum of the coefficients of the gaseous products and the sum of the coefficients of the gaseous reactants in the reaction (the change in moles of gas between the reactants and the products). For the gas-phase reaction \( mA + nB \rightleftharpoons xC + yD \), we have

\[
\Delta n = (x+y) - (m+n)
\]

**Example 13.4**

**Calculation of \( K_P \)**

Write the equations for the conversion of \( K_c \) to \( K_P \) for each of the following reactions:

(a) \( C_2H_6(g) \rightleftharpoons C_2H_4(g) + H_2(g) \)

(b) \( CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g) \)

(c) \( N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) \)

(d) \( K_c \) is equal to 0.28 for the following reaction at 900 °C:

\( CS_2(g) + 4H_2(g) \rightleftharpoons CH_4(g) + 2H_2S(g) \)

What is \( K_P \) at this temperature?

**Solution**

(a) \( \Delta n = (2) - (1) = 1 \)

\[
K_P = K_c (RT)^{\Delta n} = K_c (RT)^1 = K_c (RT)
\]

(b) \( \Delta n = (2) - (2) = 0 \)

\[
K_P = K_c (RT)^{\Delta n} = K_c (RT)^0 = K_c
\]

(c) \( \Delta n = (2) - (1 + 3) = -2 \)

\[
K_P = K_c (RT)^{\Delta n} = K_c (RT)^{-2} = \frac{K_c}{(RT)^2}
\]

(d) \( K_P = K_c (RT)^{\Delta n} = (0.28)[(0.0821)(1173)]^{-2} = 3.0 \times 10^{-5} \)

**Check Your Learning**

Write the equations for the conversion of \( K_c \) to \( K_P \) for each of the following reactions, which occur in the gas phase:
(a) \[2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)\]
(b) \[\text{N}_2\text{O}_4(g) \rightleftharpoons 2\text{NO}_2(g)\]
(c) \[\text{C}_3\text{H}_8(g) + 5\text{O}_2(g) \rightleftharpoons 3\text{CO}_2(g) + 4\text{H}_2\text{O}(g)\]
(d) At 227 °C, the following reaction has \(K_c = 0.0952:\)
\[\text{CH}_3\text{OH}(g) \rightleftharpoons \text{CO}(g) + 2\text{H}_2(g)\]
What would be the value of \(K_P\) at this temperature?
**Answer:** (a) \(K_P = K_c (RT)^{-1}\); (b) \(K_P = K_c (RT)\); (c) \(K_P = K_c (RT)\); (d) 160 or \(1.6 \times 10^2\)

### Heterogeneous Equilibria

A heterogeneous equilibrium is a system in which reactants and products are found in two or more phases. The phases may be any combination of solid, liquid, or gas phases, and solutions. When dealing with these equilibria, remember that solids and pure liquids do not appear in equilibrium constant expressions (the activities of pure solids, pure liquids, and solvents are 1).

Some heterogeneous equilibria involve chemical changes; for example:

\[\text{PbCl}_2(s) \rightleftharpoons \text{Pb}^{2+}(aq) + 2\text{Cl}^-(aq)\]
\[\text{CaO}(s) + \text{CO}_2(g) \rightleftharpoons \text{CaCO}_3(s)\]
\[\text{C}(s) + 2\text{S}(g) \rightleftharpoons \text{CS}_2(g)\]

\[K_c = [\text{Pb}^{2+}][\text{Cl}^-]^2\]
\[K_c = \frac{1}{[\text{CO}_2]}\]
\[K_c = \frac{[\text{CS}_2]}{[\text{S}]^2}\]

Other heterogeneous equilibria involve phase changes, for example, the evaporation of liquid bromine, as shown in the following equation:

\[\text{Br}_2(l) \rightleftharpoons \text{Br}_2(g)\]

\[K_c = [\text{Br}_2]\]

We can write equations for reaction quotients of heterogeneous equilibria that involve gases, using partial pressures instead of concentrations. Two examples are:

\[\text{CaO}(s) + \text{CO}_2(g) \rightleftharpoons \text{CaCO}_3(s)\]
\[K_P = \frac{1}{P_{\text{CO}_2}}\]

\[\text{C}(s) + 2\text{S}(g) \rightleftharpoons \text{CS}_2(g)\]
\[K_P = \frac{P_{\text{CS}_2}}{(P_S)^2}\]

### 13.3 Shifting Equilibria: Le Châtelier’s Principle

By the end of this section, you will be able to:
- Describe the ways in which an equilibrium system can be stressed
- Predict the response of a stressed equilibrium using Le Châtelier’s principle

As we saw in the previous section, reactions proceed in both directions (reactants go to products and products go to reactants). We can tell a reaction is at equilibrium if the reaction quotient \(Q\) is equal to the equilibrium constant \(K\). We next address what happens when a system at equilibrium is disturbed so that \(Q\) is no longer equal to \(K\). If a system at equilibrium is subjected to a perturbation or stress (such as a change in concentration) the position of equilibrium changes. Since this stress affects the concentrations of the reactants and the products, the value of \(Q\) will no longer equal the value of \(K\). To re-establish equilibrium, the system will either shift toward the products (if \(Q < K\)) or the reactants (if \(Q > K\)) until \(Q\) returns to the same value as \(K\).
This process is described by **Le Châtelier’s principle**: When a chemical system at equilibrium is disturbed, it returns to equilibrium by counteracting the disturbance. As described in the previous paragraph, the disturbance causes a change in $Q$; the reaction will shift to re-establish $Q = K$.

**Predicting the Direction of a Reversible Reaction**

Le Châtelier’s principle can be used to predict changes in equilibrium concentrations when a system that is at equilibrium is subjected to a stress. However, if we have a mixture of reactants and products that have not yet reached equilibrium, the changes necessary to reach equilibrium may not be so obvious. In such a case, we can compare the values of $Q$ and $K$ for the system to predict the changes.

**Effect of Change in Concentration on Equilibrium**

A chemical system at equilibrium can be temporarily shifted out of equilibrium by adding or removing one or more of the reactants or products. The concentrations of both reactants and products then undergo additional changes to return the system to equilibrium.

The stress on the system in **Figure 13.8** is the reduction of the equilibrium concentration of SCN$^-$ (lowering the concentration of one of the reactants would cause $Q$ to be larger than $K$). As a consequence, Le Châtelier’s principle leads us to predict that the concentration of Fe(SCN)$^{2+}$ should decrease, increasing the concentration of SCN$^-$ part way back to its original concentration, and increasing the concentration of Fe$^{3+}$ above its initial equilibrium concentration.

![Figure 13.8](credit: modification of work by Mark Ott)

The effect of a change in concentration on a system at equilibrium is illustrated further by the equilibrium of this chemical reaction:

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g) \quad K_c = 50.0 \text{ at } 400 \, ^\circ C$$

The numeric values for this example have been determined experimentally. A mixture of gases at 400 °C with $[H_2] = [I_2] = 0.221 \, M$ and $[HI] = 1.563 \, M$ is at equilibrium; for this mixture, $Q_c = K_c = 50.0$. If $H_2$ is introduced into the system so quickly that its concentration doubles before it begins to react (new $[H_2] = 0.442 \, M$), the reaction will shift so that a new equilibrium is reached, at which $[H_2] = 0.374 \, M$, $[I_2] = 0.153 \, M$, and $[HI] = 1.692 \, M$. This gives:
We have stressed this system by introducing additional H₂. The stress is relieved when the reaction shifts to the right, using up some (but not all) of the excess H₂, reducing the amount of uncombined I₂, and forming additional HI.

**Effect of Change in Pressure on Equilibrium**

Sometimes we can change the position of equilibrium by changing the pressure of a system. However, changes in pressure have a measurable effect only in systems in which gases are involved, and then only when the chemical reaction produces a change in the total number of gas molecules in the system. An easy way to recognize such a system is to look for different numbers of moles of gas on the reactant and product sides of the equilibrium. While evaluating pressure (as well as related factors like volume), it is important to remember that equilibrium constants are defined with regard to concentration (for $K_c$) or partial pressure (for $K_p$). Some changes to total pressure, like adding an inert gas that is not part of the equilibrium, will change the total pressure but not the partial pressures of the gases in the equilibrium constant expression. Thus, addition of a gas not involved in the equilibrium will not perturb the equilibrium.

As we increase the pressure of a gaseous system at equilibrium, either by decreasing the volume of the system or by adding more of one of the components of the equilibrium mixture, we introduce a stress by increasing the partial pressures of one or more of the components. In accordance with Le Châtelier’s principle, a shift in the equilibrium that reduces the total number of molecules per unit of volume will be favored because this relieves the stress. The reverse reaction would be favored by a decrease in pressure.

Consider what happens when we increase the pressure on a system in which NO, O₂, and NO₂ are at equilibrium:

$$2\text{NO}(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}_2(g)$$

The formation of additional amounts of NO₂ decreases the total number of molecules in the system because each time two molecules of NO₂ form, a total of three molecules of NO and O₂ are consumed. This reduces the total pressure exerted by the system and reduces, but does not completely relieve, the stress of the increased pressure. On the other hand, a decrease in the pressure on the system favors decomposition of NO₂ into NO and O₂, which tends to restore the pressure.

Now consider this reaction:

$$\text{N}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}(g)$$

Because there is no change in the total number of molecules in the system during reaction, a change in pressure does not favor either formation or decomposition of gaseous nitrogen monoxide.

**Effect of Change in Temperature on Equilibrium**

Changing concentration or pressure perturbs an equilibrium because the reaction quotient is shifted away from the equilibrium value. Changing the temperature of a system at equilibrium has a different effect: A change in temperature actually changes the value of the equilibrium constant. However, we can qualitatively predict the effect of the temperature change by treating it as a stress on the system and applying Le Châtelier’s principle.
When hydrogen reacts with gaseous iodine, heat is evolved.

\[ H_2(g) + I_2(g) \rightleftharpoons 2HI(g) \quad \Delta H = -9.4 \text{ kJ (exothermic)} \]

Because this reaction is exothermic, we can write it with heat as a product.

\[ H_2(g) + I_2(g) \rightleftharpoons 2HI(g) + \text{heat} \]

Increasing the temperature of the reaction increases the internal energy of the system. Thus, increasing the temperature has the effect of increasing the amount of one of the products of this reaction. The reaction shifts to the left to relieve the stress, and there is an increase in the concentration of \( H_2 \) and \( I_2 \) and a reduction in the concentration of HI. Lowering the temperature of this system reduces the amount of energy present, favors the production of heat, and favors the formation of hydrogen iodide.

When we change the temperature of a system at equilibrium, the equilibrium constant for the reaction changes. Lowering the temperature in the HI system increases the equilibrium constant: At the new equilibrium the concentration of HI has increased and the concentrations of \( H_2 \) and \( I_2 \) decreased. Raising the temperature decreases the value of the equilibrium constant, from 67.5 at 357 °C to 50.0 at 400 °C.

Temperature affects the equilibrium between \( NO_2 \) and \( N_2O_4 \) in this reaction

\[ N_2O_4(g) \rightleftharpoons 2NO_2(g) \quad \Delta H = 57.20 \text{ kJ} \]

The positive \( \Delta H \) value tells us that the reaction is endothermic and could be written

\[ \text{heat} + N_2O_4(g) \rightleftharpoons 2NO_2(g) \]

At higher temperatures, the gas mixture has a deep brown color, indicative of a significant amount of brown \( NO_2 \) molecules. If, however, we put a stress on the system by cooling the mixture (withdrawing energy), the equilibrium shifts to the left to supply some of the energy lost by cooling. The concentration of colorless \( N_2O_4 \) increases, and the concentration of brown \( NO_2 \) decreases, causing the brown color to fade.

**Catalysts Do Not Affect Equilibrium**

As we learned during our study of kinetics, a catalyst can speed up the rate of a reaction. Though this increase in reaction rate may cause a system to reach equilibrium more quickly (by speeding up the forward and reverse reactions), a catalyst has no effect on the value of an equilibrium constant nor on equilibrium concentrations.

The interplay of changes in concentration or pressure, temperature, and the lack of an influence of a catalyst on a chemical equilibrium is illustrated in the industrial synthesis of ammonia from nitrogen and hydrogen according to the equation

\[ N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) \]

A large quantity of ammonia is manufactured by this reaction. Each year, ammonia is among the top 10 chemicals, by mass, manufactured in the world. About 2 billion pounds are manufactured in the United States each year.

Ammonia plays a vital role in our global economy. It is used in the production of fertilizers and is, itself, an important fertilizer for the growth of corn, cotton, and other crops. Large quantities of ammonia are converted to nitric acid, which plays an important role in the production of fertilizers, explosives, plastics, dyes, and fibers, and is also used in the steel industry.
Fritz Haber

In the early 20th century, German chemist Fritz Haber (Figure 13.9) developed a practical process for converting diatomic nitrogen, which cannot be used by plants as a nutrient, to ammonia, a form of nitrogen that is easiest for plants to absorb.

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g) \]

The availability of nitrogen is a strong limiting factor to the growth of plants. Despite accounting for 78% of air, diatomic nitrogen (N\(_2\)) is nutritionally unavailable due to the tremendous stability of the nitrogen-nitrogen triple bond. For plants to use atmospheric nitrogen, the nitrogen must be converted to a more bioavailable form (this conversion is called nitrogen fixation).

Haber was born in Breslau, Prussia (presently Wroclaw, Poland) in December 1868. He went on to study chemistry and, while at the University of Karlsruhe, he developed what would later be known as the Haber process: the catalytic formation of ammonia from hydrogen and atmospheric nitrogen under high temperatures and pressures. For this work, Haber was awarded the 1918 Nobel Prize in Chemistry for synthesis of ammonia from its elements. The Haber process was a boon to agriculture, as it allowed the production of fertilizers to no longer be dependent on mined feed stocks such as sodium nitrate. Currently, the annual production of synthetic nitrogen fertilizers exceeds 100 million tons and synthetic fertilizer production has increased the number of humans that arable land can support from 1.9 persons per hectare in 1908 to 4.3 in 2008.

In addition to his work in ammonia production, Haber is also remembered by history as one of the fathers of chemical warfare. During World War I, he played a major role in the development of poisonous gases used for trench warfare. Regarding his role in these developments, Haber said, “During peace time a scientist belongs to the World, but during war time he belongs to his country.”[1] Haber defended the use of gas warfare against accusations that it was inhumane, saying that death was death, by whatever means it was inflicted. He stands as an example of the ethical dilemmas that face scientists in times of war and the double-edged nature of the sword of science.

Like Haber, the products made from ammonia can be multifaceted. In addition to their value for agriculture, nitrogen compounds can also be used to achieve destructive ends. Ammonium nitrate has also been used in explosives, including improvised explosive devices. Ammonium nitrate was one of the components of the bomb used in the attack on the Alfred P. Murrah Federal Building in downtown Oklahoma City on April 19, 1995.

---

It has long been known that nitrogen and hydrogen react to form ammonia. However, it became possible to manufacture ammonia in useful quantities by the reaction of nitrogen and hydrogen only in the early 20th century after the factors that influence its equilibrium were understood.

To be practical, an industrial process must give a large yield of product relatively quickly. One way to increase the yield of ammonia is to increase the pressure on the system in which \( \text{N}_2, \text{H}_2, \text{and NH}_3 \) are at equilibrium or are coming to equilibrium.

\[
\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)
\]

The formation of additional amounts of ammonia reduces the total pressure exerted by the system and somewhat reduces the stress of the increased pressure.

Although increasing the pressure of a mixture of \( \text{N}_2, \text{H}_2, \) and \( \text{NH}_3 \) will increase the yield of ammonia, at low temperatures, the rate of formation of ammonia is slow. At room temperature, for example, the reaction is so slow that if we prepared a mixture of \( \text{N}_2 \) and \( \text{H}_2 \), no detectable amount of ammonia would form during our lifetime. The formation of ammonia from hydrogen and nitrogen is an exothermic process:

\[
\text{N}_2(g) + 3\text{H}_2(g) \longrightarrow 2\text{NH}_3(g) \quad \Delta H = -92.2 \text{ kJ}
\]

Thus, increasing the temperature to increase the rate lowers the yield. If we lower the temperature to shift the equilibrium to favor the formation of more ammonia, equilibrium is reached more slowly because of the large decrease of reaction rate with decreasing temperature.

Part of the rate of formation lost by operating at lower temperatures can be recovered by using a catalyst. The net effect of the catalyst on the reaction is to cause equilibrium to be reached more rapidly.

In the commercial production of ammonia, conditions of about 500 °C, 150–900 atm, and the presence of a catalyst are used to give the best compromise among rate, yield, and the cost of the equipment necessary to produce and contain high-pressure gases at high temperatures (Figure 13.10).
13.4 Equilibrium Calculations

By the end of this section, you will be able to:

- Write equations representing changes in concentration and pressure for chemical species in equilibrium systems
- Use algebra to perform various types of equilibrium calculations

We know that at equilibrium, the value of the reaction quotient of any reaction is equal to its equilibrium constant. Thus, we can use the mathematical expression for $Q$ to determine a number of quantities associated with a reaction at equilibrium or approaching equilibrium. While we have learned to identify in which direction a reaction will shift to reach equilibrium, we want to extend that understanding to quantitative calculations. We do so by evaluating the ways that the concentrations of products and reactants change as a reaction approaches equilibrium, keeping in mind the stoichiometric ratios of the reaction. This algebraic approach to equilibrium calculations will be explored in this section.

Changes in concentrations or pressures of reactants and products occur as a reaction system approaches equilibrium. In this section we will see that we can relate these changes to each other using the coefficients in the balanced chemical equation describing the system. We use the decomposition of ammonia as an example.

On heating, ammonia reversibly decomposes into nitrogen and hydrogen according to this equation:

$$2\text{NH}_3(g) \rightleftharpoons \text{N}_2(g) + 3\text{H}_2(g)$$

If a sample of ammonia decomposes in a closed system and the concentration of $\text{N}_2$ increases by 0.11 M, the change in
the \( N_2 \) concentration, \( \Delta[N_2] \), the final concentration minus the initial concentration, is 0.11 \( M \). The change is positive because the concentration of \( N_2 \) increases.

The change in the \( H_2 \) concentration, \( \Delta[H_2] \), is also positive—the concentration of \( H_2 \) increases as ammonia decomposes. The chemical equation tells us that the change in the concentration of \( H_2 \) is three times the change in the concentration of \( N_2 \) because for each mole of \( N_2 \) produced, 3 moles of \( H_2 \) are produced.

\[
\Delta[H_2] = 3 \times \Delta[N_2] = 3 \times (0.11 \, M) = 0.33 \, M
\]

The change in concentration of \( NH_3 \), \( \Delta[NH_3] \), is twice that of \( \Delta[N_2] \); the equation indicates that 2 moles of \( NH_3 \) must decompose for each mole of \( N_2 \) formed. However, the change in the \( NH_3 \) concentration is negative because the concentration of ammonia decreases as it decomposes.

\[
\Delta[NH_3] = -2 \times \Delta[N_2] = -2 \times (0.11 \, M) = -0.22 \, M
\]

We can relate these relationships directly to the coefficients in the equation

\[
2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)
\]

The coefficients in the \( \Delta \) terms are identical to those in the balanced equation for the reaction.

The simplest way for us to find the coefficients for the concentration changes in any reaction is to use the coefficients in the balanced chemical equation. The sign of the coefficient is positive when the concentration increases; it is negative when the concentration decreases.

### Example 13.5

**Determining Relative Changes in Concentration**

Complete the changes in concentrations for each of the following reactions.

(a) \( C_2H_2(g) + 2Br_2(g) \rightleftharpoons C_2H_2\text{Br}_4(g) \)

(b) \( I_2(aq) + I^-(aq) \rightleftharpoons I_3^-(aq) \)

(c) \( C_3H_8(g) + 5O_2(g) \rightleftharpoons 3CO_2(g) + 4H_2O(g) \)

**Solution**

(a) \( x \) \( \Delta C_2H_2(g) = -x \) \( 2x \) \( \Delta C_2H_2\text{Br}_4(g) = -x \)
(b) $I_2(aq) + I^-(aq) \rightleftharpoons I_3^-(aq)\quad -x\quad -x\quad x$

(c) $C_3H_8(g) + 5O_2(g) \rightleftharpoons 3CO_2(g) + 4H_2O(g)\quad x\quad 5x\quad -3x\quad -4x$

**Check Your Learning**

Complete the changes in concentrations for each of the following reactions:

(a) $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)\quad ____\quad x\quad ____$

(b) $C_4H_8(g) \rightleftharpoons 2C_2H_4(g)\quad ____\quad -2x$

(c) $4NH_3(g) + 7H_2O(g) \rightleftharpoons 4NO_2(g) + 6H_2O(g)\quad ____\quad ____\quad ____\quad ____$

**Answer:** (a) $2x$, $x$, $-2x$; (b) $x$, $-2x$; (c) $4x$, $7x$, $-4x$, $-6x$ or $-4x$, $-7x$, $4x$, $6x$

**Calculations Involving Equilibrium Concentrations**

Because the value of the reaction quotient of any reaction at equilibrium is equal to its equilibrium constant, we can use the mathematical expression for $Q_c$ (i.e., the law of mass action) to determine a number of quantities associated with a reaction at equilibrium. It may help if we keep in mind that $Q_c = K_c$ (at equilibrium) in all of these situations and that there are only three basic types of calculations:

1. **Calculation of an equilibrium constant.** If concentrations of reactants and products at equilibrium are known, the value of the equilibrium constant for the reaction can be calculated.

2. **Calculation of missing equilibrium concentrations.** If the value of the equilibrium constant and all of the equilibrium concentrations, except one, are known, the remaining concentration can be calculated.

3. **Calculation of equilibrium concentrations from initial concentrations.** If the value of the equilibrium constant and a set of concentrations of reactants and products that are not at equilibrium are known, the concentrations at equilibrium can be calculated.

A similar list could be generated using $Q_P$, $K_P$, and partial pressure. We will look at solving each of these cases in sequence.

**Calculation of an Equilibrium Constant**

Since the law of mass action is the only equation we have to describe the relationship between $K_c$ and the concentrations of reactants and products, any problem that requires us to solve for $K_c$ must provide enough information to determine the reactant and product concentrations at equilibrium. Armed with the concentrations, we can solve the equation for $K_c$, as it will be the only unknown.

**Example 13.6** showed us how to determine the equilibrium constant of a reaction if we know the concentrations of reactants and products at equilibrium. The following example shows how to use the stoichiometry of the reaction and a combination of initial concentrations and equilibrium concentrations to determine an equilibrium constant. This technique, commonly called an ICE chart—for Initial, Change, and Equilibrium—will be helpful in solving many equilibrium problems. A chart is generated beginning with the equilibrium reaction in question. Underneath the reaction the initial concentrations of the reactants and products are listed—these conditions are usually provided in the problem and we consider no shift toward equilibrium to have happened. The next row of data is the change that occurs as the system shifts toward equilibrium—do not forget to consider the reaction stoichiometry as described in a previous section of this chapter. The last row contains the concentrations once equilibrium has been reached.
Example 13.6

Calculation of an Equilibrium Constant

Iodine molecules react reversibly with iodide ions to produce triiodide ions.

\[ I_2(aq) + I^-(aq) \rightleftharpoons I_3^-(aq) \]

If a solution with the concentrations of \( I_2 \) and \( I^- \) both equal to \( 1.000 \times 10^{-3} \) M before reaction gives an equilibrium concentration of \( I_2 \) of \( 6.61 \times 10^{-4} \) M, what is the equilibrium constant for the reaction?

**Solution**

We will begin this problem by calculating the changes in concentration as the system goes to equilibrium. Then we determine the equilibrium concentrations and, finally, the equilibrium constant. First, we set up a table with the initial concentrations, the changes in concentrations, and the equilibrium concentrations using \(-x\) as the change in concentration of \( I_2 \).

<table>
<thead>
<tr>
<th></th>
<th>( I_2 )</th>
<th>( I^- )</th>
<th>( I_3^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial concentration (M)</strong></td>
<td>( 1.000 \times 10^{-3} )</td>
<td>( 1.000 \times 10^{-3} )</td>
<td>0</td>
</tr>
<tr>
<td><strong>Change (M)</strong></td>
<td>(-x)</td>
<td>(-x)</td>
<td>(+x)</td>
</tr>
<tr>
<td><strong>Equilibrium concentration (M)</strong></td>
<td>( 1.000 \times 10^{-3} - x )</td>
<td>( 1.000 \times 10^{-3} - x )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

Since the equilibrium concentration of \( I_2 \) is given, we can solve for \( x \). At equilibrium the concentration of \( I_2 \) is \( 6.61 \times 10^{-4} \) M so that

\[
1.000 \times 10^{-3} - x = 6.61 \times 10^{-4}
\]

\[
x = 1.000 \times 10^{-3} - 6.61 \times 10^{-4}
\]

\[
= 3.39 \times 10^{-4} \text{ M}
\]

Now we can fill in the table with the concentrations at equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>( I_2 )</th>
<th>( I^- )</th>
<th>( I_3^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial concentration (M)</strong></td>
<td>( 1.000 \times 10^{-3} )</td>
<td>( 1.000 \times 10^{-3} )</td>
<td>0</td>
</tr>
<tr>
<td><strong>Change (M)</strong></td>
<td>(-x = 3.39 \times 10^{-4})</td>
<td>(-x)</td>
<td>(+x)</td>
</tr>
<tr>
<td><strong>Equilibrium concentration (M)</strong></td>
<td>( 6.61 \times 10^{-4} )</td>
<td>( 6.61 \times 10^{-4} )</td>
<td>( 3.39 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

We now calculate the value of the equilibrium constant.

\[
K_c = Q_c = \frac{[I_3^-]}{[I_2][I^-]} = \frac{3.39 \times 10^{-4} \text{ M}}{(6.61 \times 10^{-4} \text{ M})(6.61 \times 10^{-4} \text{ M})} = 776
\]

**Check Your Learning**

Ethanol and acetic acid react and form water and ethyl acetate, the solvent responsible for the odor of some nail polish removers.

\[ C_2H_5OH + CH_3CO_2H \rightleftharpoons CH_3CO_2C_2H_5 + H_2O \]

When 1 mol each of \( C_2H_5OH \) and \( CH_3CO_2H \) are allowed to react in 1 L of the solvent dioxane, equilibrium is established when \( \frac{1}{3} \) mol of each of the reactants remains. Calculate the equilibrium constant for the reaction.
reaction. (Note: Water is not a solvent in this reaction.)

Answer: $K_c = 4$

Calculation of a Missing Equilibrium Concentration

If we know the equilibrium constant for a reaction and know the concentrations at equilibrium of all reactants and products except one, we can calculate the missing concentration.

Example 13.7

Calculation of a Missing Equilibrium Concentration

Nitrogen oxides are air pollutants produced by the reaction of nitrogen and oxygen at high temperatures. At 2000 °C, the value of the equilibrium constant for the reaction, $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$, is $4.1 \times 10^{-4}$. Find the concentration of NO(g) in an equilibrium mixture with air at 1 atm pressure at this temperature. In air, $[N_2] = 0.036 \text{ mol/L}$ and $[O_2] = 0.0089 \text{ mol/L}$.

Solution

We are given all of the equilibrium concentrations except that of NO. Thus, we can solve for the missing equilibrium concentration by rearranging the equation for the equilibrium constant.

$$K_c = Q_c = \frac{[\text{NO}]^2}{[\text{N}_2][\text{O}_2]}$$

$$[\text{NO}]^2 = K_c [\text{N}_2][\text{O}_2]$$

$$[\text{NO}] = \sqrt{K_c [\text{N}_2][\text{O}_2]}$$

$$= \sqrt{(4.1 \times 10^{-4})(0.036)(0.0089)}$$

$$= \sqrt{1.31 \times 10^{-7}}$$

$$= 3.6 \times 10^{-4}$$

Thus $[\text{NO}]$ is $3.6 \times 10^{-4} \text{ mol/L}$ at equilibrium under these conditions.

We can check our answer by substituting all equilibrium concentrations into the expression for the reaction quotient to see whether it is equal to the equilibrium constant.

$$Q_c = \frac{[\text{NO}]^2}{[\text{N}_2][\text{O}_2]}$$

$$= \frac{(3.6 \times 10^{-4})^2}{(0.036)(0.0089)}$$

$$Q_c = 4.0 \times 10^{-4} = K_c$$

The answer checks; our calculated value gives the equilibrium constant within the error associated with the significant figures in the problem.

Check Your Learning

The equilibrium constant for the reaction of nitrogen and hydrogen to produce ammonia at a certain temperature is $6.00 \times 10^{-2}$. Calculate the equilibrium concentration of ammonia if the equilibrium concentrations of nitrogen and hydrogen are $4.26 \text{ M}$ and $2.09 \text{ M}$, respectively.

Answer: $1.53 \text{ mol/L}$

Calculation of Changes in Concentration

If we know the equilibrium constant for a reaction and a set of concentrations of reactants and products that are not
at equilibrium, we can calculate the changes in concentrations as the system comes to equilibrium, as well as the new concentrations at equilibrium. The typical procedure can be summarized in four steps.

1. Determine the direction the reaction proceeds to come to equilibrium.
   a. Write a balanced chemical equation for the reaction.
   b. If the direction in which the reaction must proceed to reach equilibrium is not obvious, calculate $Q_c$ from the initial concentrations and compare to $K_c$ to determine the direction of change.

2. Determine the relative changes needed to reach equilibrium, then write the equilibrium concentrations in terms of these changes.
   a. Define the changes in the initial concentrations that are needed for the reaction to reach equilibrium. Generally, we represent the smallest change with the symbol $x$ and express the other changes in terms of the smallest change.
   b. Define missing equilibrium concentrations in terms of the initial concentrations and the changes in concentration determined in (a).

3. Solve for the change and the equilibrium concentrations.
   a. Substitute the equilibrium concentrations into the expression for the equilibrium constant, solve for $x$, and check any assumptions used to find $x$.
   b. Calculate the equilibrium concentrations.

4. Check the arithmetic.
   a. Check the calculated equilibrium concentrations by substituting them into the equilibrium expression and determining whether they give the equilibrium constant.
   Sometimes a particular step may differ from problem to problem—it may be more complex in some problems and less complex in others. However, every calculation of equilibrium concentrations from a set of initial concentrations will involve these steps.
   In solving equilibrium problems that involve changes in concentration, sometimes it is convenient to set up an ICE table, as described in the previous section.

**Example 13.8**

**Calculation of Concentration Changes as a Reaction Goes to Equilibrium**

Under certain conditions, the equilibrium constant for the decomposition of $\text{PCl}_5(g)$ into $\text{PCl}_3(g)$ and $\text{Cl}_2(g)$ is $0.0211$. What are the equilibrium concentrations of $\text{PCl}_5$, $\text{PCl}_3$, and $\text{Cl}_2$ if the initial concentration of $\text{PCl}_5$ was 1.00 M?

**Solution**

Use the stepwise process described earlier.

**Step 1.** Determine the direction the reaction proceeds.

The balanced equation for the decomposition of $\text{PCl}_5$ is

$$\text{PCl}_5(g) \rightleftharpoons \text{PCl}_3(g) + \text{Cl}_2(g)$$

Because we have no products initially, $Q_c = 0$ and the reaction will proceed to the right.

**Step 2.** Determine the relative changes needed to reach equilibrium, then write the equilibrium concentrations in terms of these changes.

Let us represent the increase in concentration of $\text{PCl}_3$ by the symbol $x$. The other changes may be written in terms of $x$ by considering the coefficients in the chemical equation.

$$\text{PCl}_5(g) \rightleftharpoons \text{PCl}_3(g) + \text{Cl}_2(g)$$

$$-x \quad x \quad x$$
The changes in concentration and the expressions for the equilibrium concentrations are:

<table>
<thead>
<tr>
<th></th>
<th>PCl₅</th>
<th>PCl₃</th>
<th>+</th>
<th>Cl₂</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial concentration (M)</strong></td>
<td>1.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Change (M)</strong></td>
<td>−x</td>
<td>+x</td>
<td>+x</td>
<td></td>
</tr>
<tr>
<td><strong>Equilibrium concentration (M)</strong></td>
<td>1.00 − x</td>
<td>0 + x = x</td>
<td>0 + x = x</td>
<td></td>
</tr>
</tbody>
</table>

**Step 3. Solve for the change and the equilibrium concentrations.**

Substituting the equilibrium concentrations into the equilibrium constant equation gives

\[
K_c = \frac{[PCl_3][Cl_2]}{[PCl_5]} = 0.0211
\]

\[
= \frac{(x)(x)}{(1.00 - x)}
\]

This equation contains only one variable, \(x\), the change in concentration. We can write the equation as a quadratic equation and solve for \(x\) using the quadratic formula.

\[
0.0211 = \frac{(x)(x)}{(1.00 - x)}
\]

\[
x^2 + 0.0211x - 0.0211 = 0
\]

Appendix B shows us an equation of the form \(ax^2 + bx + c = 0\) can be rearranged to solve for \(x\):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

In this case, \(a = 1\), \(b = 0.0211\), and \(c = -0.0211\). Substituting the appropriate values for \(a\), \(b\), and \(c\) yields:

\[
x = \frac{-0.0211 \pm \sqrt{(0.0211)^2 - 4(1)(-0.0211)}}{2(1)}
\]

\[
= \frac{-0.0211 \pm \sqrt{(4.45 \times 10^{-4}) + (8.44 \times 10^{-2})}}{2}
\]

\[
= \frac{-0.0211 \pm 0.291}{2}
\]

Hence

\[
x = \frac{-0.0211 + 0.291}{2} = 0.135
\]

or

\[
x = \frac{-0.0211 - 0.291}{2} = -0.156
\]

Quadratic equations often have two different solutions, one that is physically possible and one that is physically impossible (an extraneous root). In this case, the second solution (−0.156) is physically impossible because we know the change must be a positive number (otherwise we would end up with negative values for concentrations of the products). Thus, \(x = 0.135\) M.

The equilibrium concentrations are

\[
[PCl_5] = 1.00 - 0.135 = 0.87\ M
\]

\[
[PCl_3] = x = 0.135\ M
\]

\[
[Cl_2] = x = 0.135\ M
\]

**Step 4. Check the arithmetic.**

Substitution into the expression for \(K_c\) (to check the calculation) gives
The equilibrium constant calculated from the equilibrium concentrations is equal to the value of $K_c$ given in the problem (when rounded to the proper number of significant figures). Thus, the calculated equilibrium concentrations check.

### Check Your Learning

Acetic acid, CH$_3$CO$_2$H, reacts with ethanol, C$_2$H$_5$OH, to form water and ethyl acetate, CH$_3$CO$_2$C$_2$H$_5$.

$$\text{CH}_3\text{CO}_2\text{H} + \text{C}_2\text{H}_5\text{OH} \rightleftharpoons \text{CH}_3\text{CO}_2\text{C}_2\text{H}_5 + \text{H}_2\text{O}$$

The equilibrium constant for this reaction with dioxane as a solvent is 4.0. What are the equilibrium concentrations when a mixture that is 0.15 M in CH$_3$CO$_2$H, 0.15 M in C$_2$H$_5$OH, 0.40 M in CH$_3$CO$_2$C$_2$H$_5$, and 0.40 M in H$_2$O are mixed in enough dioxane to make 1.0 L of solution?

**Answer:** [CH$_3$CO$_2$H] = 0.36 M, [C$_2$H$_5$OH] = 0.36 M, [CH$_3$CO$_2$C$_2$H$_5$] = 0.17 M, [H$_2$O] = 0.17 M

### Check Your Learning

A 1.00-L flask is filled with 1.00 moles of H$_2$ and 2.00 moles of I$_2$. The value of the equilibrium constant for the reaction of hydrogen and iodine reacting to form hydrogen iodide is 50.5 under the given conditions. What are the equilibrium concentrations of H$_2$, I$_2$, and HI in moles/L?

**Answer:** [H$_2$] = 0.06 M, [I$_2$] = 1.06 M, [HI] = 1.88 M

Sometimes it is possible to use chemical insight to find solutions to equilibrium problems without actually solving a quadratic (or more complicated) equation. First, however, it is useful to verify that equilibrium can be obtained starting from two extremes: all (or mostly) reactants and all (or mostly) products (similar to what was shown in Figure 13.7).

Consider the ionization of 0.150 M HA, a weak acid.

$$\text{HA}(aq) \rightleftharpoons \text{H}^+(aq) + \text{A}^- (aq) \quad K_c = 6.80 \times 10^{-4}$$

The most obvious way to determine the equilibrium concentrations would be to start with only reactants. This could be called the “all reactant” starting point. Using $x$ for the amount of acid ionized at equilibrium, this is the ICE table and solution.

<table>
<thead>
<tr>
<th>Initial concentration (M)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HA(aq)</td>
<td>0.150</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Change (M)</td>
<td>$-x$</td>
<td>$x$</td>
<td>$x$</td>
</tr>
<tr>
<td>Equilibrium concentration (M)</td>
<td>0.150 $-x$</td>
<td>$x$</td>
<td>$x$</td>
</tr>
</tbody>
</table>

Setting up and solving the quadratic equation gives

$$K_c = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} = \frac{(x)(x)}{(0.150 - x)} = 6.80 \times 10^{-4}$$

$$x^2 + 6.80 \times 10^{-4}x - 1.02 \times 10^{-4} = 0$$

$$x = \frac{-6.80 \times 10^{-4} \pm \sqrt{(6.80 \times 10^{-4})^2 - (4)(1)(-1.02 \times 10^{-4})}}{2(1)}$$

$$x = 0.00977 \text{ M or } -0.0104 \text{ M}$$

Using the positive (physical) root, the equilibrium concentrations are

$$[\text{HA}] = 0.150 - x = 0.140 \text{ M}$$
\[ [H^+] = [A^-] = x = 0.00977 \text{ M} \]

A less obvious way to solve the problem would be to assume all the HA ionizes first, then the system comes to equilibrium. This could be called the “all product” starting point. Assuming all of the HA ionizes gives

\[
[\text{HA}] = 0.150 - 0.150 = 0 \text{ M} \\
[H^+] = 0 + 0.150 = 0.150 \text{ M} \\
[A^-] = 0 + 0.150 = 0.150 \text{ M}
\]

Using these as initial concentrations and “y” to represent the concentration of HA at equilibrium, this is the ICE table for this starting point.

<table>
<thead>
<tr>
<th>Initial concentration (M)</th>
<th>HA(aq)</th>
<th>H^+(aq)</th>
<th>A^-(aq)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.150</td>
<td>0.150</td>
</tr>
<tr>
<td>Change (M)</td>
<td>+y</td>
<td>-y</td>
<td>-y</td>
</tr>
<tr>
<td>Equilibrium concentration (M)</td>
<td>y</td>
<td>0.150 − y</td>
<td>0.150 − y</td>
</tr>
</tbody>
</table>

Setting up and solving the quadratic equation gives

\[
K_c = \frac{[H^+][A^-]}{[\text{HA}]} = \frac{(0.150 - y)(0.150 - y)}{y} = 6.80 \times 10^{-4} \\
6.80 \times 10^{-4}y = 0.0225 - 0.300y + y^2
\]

Retain a few extra significant figures to minimize rounding problems.

\[
y^2 - 0.30068y + 0.022500 = 0 \\
y = \frac{-0.30068 \pm \sqrt{(0.30068)^2 - (4)(1)(0.022500)}}{2(1)} \\
y = \frac{0.30068 \pm 0.020210}{2}
\]

Rounding each solution to three significant figures gives

\[ y = 0.160 \text{ M} \quad \text{or} \quad y = 0.140 \text{ M} \]

Using the physically significant root (0.140 M) gives the equilibrium concentrations as

\[
[\text{HA}] = y = 0.140 \text{ M} \\
[H^+] = 0.150 - y = 0.010 \text{ M} \\
[A^-] = 0.150 - y = 0.010 \text{ M}
\]

Thus, the two approaches give the same results (to three decimal places), and show that both starting points lead to the same equilibrium conditions. The “all reactant” starting point resulted in a relatively small change (x) because the system was close to equilibrium, while the “all product” starting point had a relatively large change (y) that was nearly the size of the initial concentrations. It can be said that a system that starts “close” to equilibrium will require only a "small" change in conditions (x) to reach equilibrium.

Recall that a small \( K_c \) means that very little of the reactants form products and a large \( K_c \) means that most of the reactants form products. If the system can be arranged so it starts “close” to equilibrium, then if the change (x) is small compared to any initial concentrations, it can be neglected. Small is usually defined as resulting in an error of less than 5%. The following two examples demonstrate this.
Example 13.9

Approximate Solution Starting Close to Equilibrium

What are the concentrations at equilibrium of a 0.15 \( M \) solution of HCN?

\[
\text{HCN}(aq) \rightleftharpoons \text{H}^+(aq) + \text{CN}^-(aq) \quad K_c = 4.9 \times 10^{-10}
\]

Solution

Using “\( x \)” to represent the concentration of each product at equilibrium gives this ICE table.

<table>
<thead>
<tr>
<th></th>
<th>HCN(aq)</th>
<th>H^+(aq)</th>
<th>+</th>
<th>CN^-(aq)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial concentration (M)</td>
<td>0.15</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Change (M)</td>
<td>(-x)</td>
<td>(x)</td>
<td></td>
<td>(x)</td>
</tr>
<tr>
<td>Equilibrium concentration (M)</td>
<td>0.15 - (x)</td>
<td>(x)</td>
<td>(x)</td>
<td></td>
</tr>
</tbody>
</table>

The exact solution may be obtained using the quadratic formula with

\[
K_c = \frac{(x)(x)}{0.15 - x}
\]

solving

\[
x^2 + 4.9 \times 10^{-10} - 7.35 \times 10^{-11} = 0
\]

\[
x = 8.56 \times 10^{-6} \text{ M (3 sig. figs ) = } 8.6 \times 10^{-6} \text{ M (2 sig. figs )}
\]

Thus \([\text{H}^+] = [\text{CN}^-] = x = 8.6 \times 10^{-6} \text{ M}\) and \([\text{HCN}] = 0.15 - x = 0.15 \text{ M}\).

In this case, chemical intuition can provide a simpler solution. From the equilibrium constant and the initial conditions, \( x \) must be small compared to 0.15 \( M \). More formally, if \( x \ll 0.15 \), then \( 0.15 - x \approx 0.15 \). If this assumption is true, then it simplifies obtaining \( x \)

\[
K_c = \frac{(x)(x)}{0.15 - x} \approx \frac{x^2}{0.15}
\]

\[
4.9 \times 10^{-10} = \frac{x^2}{0.15}
\]

\[
x^2 = (0.15)(4.9 \times 10^{-10}) = 7.4 \times 10^{-11}
\]

\[
x = \sqrt{7.4 \times 10^{-11}} = 8.6 \times 10^{-6} \text{ M}
\]

In this example, solving the exact (quadratic) equation and using approximations gave the same result to two significant figures. While most of the time the approximation is a bit different from the exact solution, as long as the error is less than 5%, the approximate solution is considered valid. In this problem, the 5% applies to \( (0.15 - x) \approx 0.15 \text{ M} \), so if

\[
\frac{x}{0.15} \times 100\% = \frac{8.6 \times 10^{-6}}{0.15} \times 100\% = 0.006\%
\]

is less than 5%, as it is in this case, the assumption is valid. The approximate solution is thus a valid solution.

Check Your Learning

What are the equilibrium concentrations in a 0.25 \( M \) \( \text{NH}_3 \) solution?

\[
\text{NH}_3(aq) + \text{H}_2\text{O}(l) \rightleftharpoons \text{NH}_4^+(aq) + \text{OH}^-(aq) \quad K_c = 1.8 \times 10^{-5}
\]

Assume that \( x \) is much less than 0.25 \( M \) and calculate the error in your assumption.

Answer: \([\text{OH}^-] = [\text{NH}_4^+] = 0.0021 \text{ M}, \ [\text{NH}_3] = 0.25 \text{ M}, \text{ error } = 0.84\%\)
The second example requires that the original information be processed a bit, but it still can be solved using a small \( x \) approximation.

**Example 13.10**

**Approximate Solution After Shifting Starting Concentration**

Copper(II) ions form a complex ion in the presence of ammonia

\[
\text{Cu}^{2+}(aq) + 4\text{NH}_3(aq) \rightleftharpoons \text{Cu(NH}_3)_4^{2+}(aq) \quad K_c = 5.0 \times 10^{13} = \frac{[\text{Cu(NH}_3)_4^{2+}]}{[\text{Cu}^{2+}(aq)][\text{NH}_3]^4}
\]

If 0.010 mol Cu\(^{2+}\) is added to 1.00 L of a solution that is 1.00 M NH\(_3\) what are the concentrations when the system comes to equilibrium?

**Solution**

The initial concentration of copper(II) is 0.010 M. The equilibrium constant is very large so it would be better to start with as much product as possible because “all products” is much closer to equilibrium than “all reactants.” Note that Cu\(^{2+}\) is the limiting reactant; if all 0.010 M of it reacts to form product the concentrations would be

\[
[\text{Cu}^{2+}] = 0.010 - 0.010 = 0 \text{ M}
\]

\[
[\text{Cu(NH}_3)_4^{2+}] = 0.010 \text{ M}
\]

\[
[\text{NH}_3] = 1.00 - 4 \times 0.010 = 0.96 \text{ M}
\]

Using these “shifted” values as initial concentrations with \( x \) as the free copper(II) ion concentration at equilibrium gives this ICE table.

<table>
<thead>
<tr>
<th>( \text{Cu}^{2+}(aq) )</th>
<th>( + )</th>
<th>( 4\text{NH}_3(aq) )</th>
<th>( \rightleftharpoons )</th>
<th>( \text{Cu(NH}_3)_4^{2+}(aq) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial concentration (M)</strong></td>
<td>0</td>
<td>0.96</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td><strong>Change (M)</strong></td>
<td>( +x )</td>
<td>( +4x )</td>
<td>( -x )</td>
<td></td>
</tr>
<tr>
<td><strong>Equilibrium concentration (M)</strong></td>
<td>( x )</td>
<td>( 0.96 + 4x )</td>
<td>( 0.010 - x )</td>
<td></td>
</tr>
</tbody>
</table>

Since we are starting close to equilibrium, \( x \) should be small so that

\[
0.96 + 4x \approx 0.96 \text{ M}
\]

\[
0.010 - x \approx 0.010 \text{ M}
\]

\[
K_c = \frac{(0.010 - x)^4}{x(0.96 - 4x)^4} \approx \frac{(0.010)}{x(0.96)^4} = 5.0 \times 10^{13}
\]

\[
x = \frac{(0.010)}{K_c(0.96)^4} = 2.4 \times 10^{-16} \text{ M}
\]

Select the smallest concentration for the 5% rule.

\[
\frac{2.4 \times 10^{-16}}{0.010} \times 100\% = 2 \times 10^{-12} \%
\]

This is much less than 5%, so the assumptions are valid. The concentrations at equilibrium are

\[
[\text{Cu}^{2+}] = x = 2.4 \times 10^{-16} \text{ M}
\]

\[
[\text{NH}_3] = 0.96 - 4x = 0.96 \text{ M}
\]

\[
[\text{Cu(NH}_3)_4^{2+}] = 0.010 - x = 0.010 \text{ M}
\]

By starting with the maximum amount of product, this system was near equilibrium and the change (\( x \)) was
very small. With only a small change required to get to equilibrium, the equation for $x$ was greatly simplified and gave a valid result well within the 5% error maximum.

**Check Your Learning**

What are the equilibrium concentrations when 0.25 mol Ni\(^{2+}\) is added to 1.00 L of 2.00 M NH\(_3\) solution?

\[
\text{Ni}^{2+}(aq) + 6\text{NH}_3(aq) \rightleftharpoons \text{Ni(NH}_3)_6^{2+}(aq) \quad K_c = 5.5 \times 10^8
\]

With such a large equilibrium constant, first form as much product as possible, then assume that only a small amount ($x$) of the product shifts left. Calculate the error in your assumption.

**Answer:** $[\text{Ni(NH}_3)_6^{2+}] = 0.25 \text{ M}$, $[\text{NH}_3] = 0.50 \text{ M}$, $[\text{Ni}^{2+}] = 2.9 \times 10^{-8} \text{ M}$, error $= 1.2 \times 10^{-5}\%$
Key Terms

**equilibrium** in chemical reactions, the state in which the conversion of reactants into products and the conversion of products back into reactants occur simultaneously at the same rate; state of balance

**equilibrium constant (K)** value of the reaction quotient for a system at equilibrium

**heterogeneous equilibria** equilibria between reactants and products in different phases

**homogeneous equilibria** equilibria within a single phase

**K_c** equilibrium constant for reactions based on concentrations of reactants and products

**K_P** equilibrium constant for gas-phase reactions based on partial pressures of reactants and products

**law of mass action** when a reversible reaction has attained equilibrium at a given temperature, the reaction quotient remains constant

**Le Châtelier’s principle** when a chemical system at equilibrium is disturbed, it returns to equilibrium by counteracting the disturbance

**position of equilibrium** concentrations or partial pressures of components of a reaction at equilibrium (commonly used to describe conditions before a disturbance)

**reaction quotient (Q)** ratio of the product of molar concentrations (or pressures) of the products to that of the reactants, each concentration (or pressure) being raised to the power equal to the coefficient in the equation

**reversible reaction** chemical reaction that can proceed in both the forward and reverse directions under given conditions

**stress** change to a reaction's conditions that may cause a shift in the equilibrium

Key Equations

\[ Q = \frac{[C]^x[D]^y}{[A]^m[B]^n} \quad \text{where } mA + nB \rightleftharpoons xC + yD \]

\[ Q_P = \frac{(P_C)^x(P_D)^y}{(P_A)^m(P_B)^n} \quad \text{where } mA + nB \rightleftharpoons xC + yD \]

\[ P = MRT \]

\[ K_P = K_c (RT)^{\Delta n} \]

Summary

13.1 Chemical Equilibria

A reaction is at equilibrium when the amounts of reactants or products no longer change. Chemical equilibrium is a dynamic process, meaning the rate of formation of products by the forward reaction is equal to the rate at which the products re-form reactants by the reverse reaction.

13.2 Equilibrium Constants

For any reaction that is at equilibrium, the reaction quotient Q is equal to the equilibrium constant K for the reaction. If a reactant or product is a pure solid, a pure liquid, or the solvent in a dilute solution, the concentration of this component does not appear in the expression for the equilibrium constant. At equilibrium, the values of the concentrations of the reactants and products are constant. Their particular values may vary depending on conditions, but the value of the reaction quotient will always equal K (K_c when using concentrations or K_P when using partial
pressures).

A homogeneous equilibrium is an equilibrium in which all components are in the same phase. A heterogeneous equilibrium is an equilibrium in which components are in two or more phases. We can decide whether a reaction is at equilibrium by comparing the reaction quotient with the equilibrium constant for the reaction.

### 13.3 Shifting Equilibria: Le Châtelier’s Principle

Systems at equilibrium can be disturbed by changes to temperature, concentration, and, in some cases, volume and pressure; volume and pressure changes will disturb equilibrium if the number of moles of gas is different on the reactant and product sides of the reaction. The system's response to these disturbances is described by Le Châtelier’s principle: The system will respond in a way that counteracts the disturbance. Not all changes to the system result in a disturbance of the equilibrium. Adding a catalyst affects the rates of the reactions but does not alter the equilibrium, and changing pressure or volume will not significantly disturb systems with no gases or with equal numbers of moles of gas on the reactant and product side.

### Effects of Disturbances of Equilibrium and \( K \)

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Observed Change as Equilibrium is Restored</th>
<th>Direction of Shift</th>
<th>Effect on ( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>reactant added</td>
<td>added reactant is partially consumed</td>
<td>toward products</td>
<td>none</td>
</tr>
<tr>
<td>product added</td>
<td>added product is partially consumed</td>
<td>toward reactants</td>
<td>none</td>
</tr>
<tr>
<td>decrease in volume/ increase in gas pressure</td>
<td>pressure decreases</td>
<td>toward side with fewer moles of gas</td>
<td>none</td>
</tr>
<tr>
<td>increase in volume/ decrease in gas pressure</td>
<td>pressure increases</td>
<td>toward side with more moles of gas</td>
<td>none</td>
</tr>
<tr>
<td>temperature increase</td>
<td>heat is absorbed</td>
<td>toward products for endothermic, toward reactants for exothermic</td>
<td>changes</td>
</tr>
<tr>
<td>temperature decrease</td>
<td>heat is given off</td>
<td>toward reactants for endothermic, toward products for exothermic</td>
<td>changes</td>
</tr>
</tbody>
</table>

**Table 13.1**

### 13.4 Equilibrium Calculations

The ratios of the rate of change in concentrations of a reaction are equal to the ratios of the coefficients in the balanced chemical equation. The sign of the coefficient of \( X \) is positive when the concentration increases and negative when it decreases. We learned to approach three basic types of equilibrium problems. When given the concentrations of the reactants and products at equilibrium, we can solve for the equilibrium constant; when given the equilibrium constant and some of the concentrations involved, we can solve for the missing concentrations; and when given the equilibrium constant and the initial concentrations, we can solve for the concentrations at equilibrium.

### Exercises

**13.1 Chemical Equilibria**

1. What does it mean to describe a reaction as “reversible”?
2. When writing an equation, how is a reversible reaction distinguished from a nonreversible reaction?
3. If a reaction is reversible, when can it be said to have reached equilibrium?
4. Is a system at equilibrium if the rate constants of the forward and reverse reactions are equal?
5. If the concentrations of products and reactants are equal, is the system at equilibrium?

13.2 Equilibrium Constants

6. Explain why there may be an infinite number of values for the reaction quotient of a reaction at a given temperature but there can be only one value for the equilibrium constant at that temperature.
7. Explain why an equilibrium between Br\(_2\)(l) and Br\(_2\)(g) would not be established if the container were not a closed vessel shown in Figure 13.5.
8. If you observe the following reaction at equilibrium, is it possible to tell whether the reaction started with pure NO\(_2\) or with pure N\(_2\)O\(_4\)?
   \[2\text{NO}_2(g) \rightleftharpoons \text{N}_2\text{O}_4(g)\]
9. Among the solubility rules previously discussed is the statement: All chlorides are soluble except Hg\(_2\)Cl\(_2\), AgCl, PbCl\(_2\), and CuCl.
   (a) Write the expression for the equilibrium constant for the reaction represented by the equation
   \[\text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq)\] Is \(K_c > 1, < 1, \) or \(\approx 1\)? Explain your answer.
   (b) Write the expression for the equilibrium constant for the reaction represented by the equation
   \[\text{Pb}^{2+}(aq) + 2\text{Cl}^{-}(aq) \rightleftharpoons \text{PbCl}_2(s).\] Is \(K_c > 1, < 1, \) or \(\approx 1\)? Explain your answer.
10. Among the solubility rules previously discussed is the statement: Carbonates, phosphates, borates, and arsenates—except those of the ammonium ion and the alkali metals—are insoluble.
   (a) Write the expression for the equilibrium constant for the reaction represented by the equation
   \[\text{CaCO}_3(s) \rightleftharpoons \text{Ca}^{2+}(aq) + \text{CO}_3^{2-}(aq).\] Is \(K_c > 1, < 1, \) or \(\approx 1\)? Explain your answer.
   (b) Write the expression for the equilibrium constant for the reaction represented by the equation
   \[3\text{Ba}^{2+}(aq) + 2\text{PO}_4^{3-}(aq) \rightleftharpoons \text{Ba}_3(\text{PO}_4)_2(s).\] Is \(K_c > 1, < 1, \) or \(\approx 1\)? Explain your answer.
11. Benzene is one of the compounds used as octane enhancers in unleaded gasoline. It is manufactured by the catalytic conversion of acetylene to benzene: \(3\text{C}_2\text{H}_2(g) \rightarrow \text{C}_6\text{H}_6(g).\) Which value of \(K_c\) would make this reaction most useful commercially? \(K_c \approx 0.01, \) \(K_c \approx 1, \) or \(K_c \approx 10.\) Explain your answer.
12. Show that the complete chemical equation, the total ionic equation, and the net ionic equation for the reaction represented by the equation \(\text{KI}(aq) + \text{I}_2(aq) \rightleftharpoons \text{KI}_3(aq)\) give the same expression for the reaction quotient. \(\text{KI}_3\) is composed of the ions \(\text{K}^+\) and \(\text{I}_3^-\).
13. For a titration to be effective, the reaction must be rapid and the yield of the reaction must essentially be 100%. Is \(K_c > 1, < 1, \) or \(\approx 1\) for a titration reaction?
14. For a precipitation reaction to be useful in a gravimetric analysis, the product of the reaction must be insoluble. Is \(K_c > 1, < 1, \) or \(\approx 1\) for a useful precipitation reaction?
15. Write the mathematical expression for the reaction quotient, \(Q_c\), for each of the following reactions:
   (a) \(\text{CH}_4(g) + \text{Cl}_2(g) \rightleftharpoons \text{CH}_3\text{Cl}(g) + \text{HCl}(g)\)
   (b) \(\text{N}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}(g)\)
   (c) \(2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)\)
   (d) \(\text{BaSO}_3(s) \rightleftharpoons \text{BaO}(s) + \text{SO}_2(g)\)
   (e) \(\text{P}_4(g) + 5\text{O}_2(g) \rightleftharpoons \text{P}_4\text{O}_{10}(s)\)
   (f) \(\text{Br}_2(g) \rightleftharpoons 2\text{Br}(g)\)
   (g) \(\text{CH}_4(g) + 2\text{O}_2(g) \rightleftharpoons \text{CO}_2(g) + 2\text{H}_2\text{O}(l)\)
   (h) \(\text{CuSO}_4 \cdot 5\text{H}_2\text{O}(s) \rightleftharpoons \text{CuSO}_4(s) + 5\text{H}_2\text{O}(g)\)
16. Write the mathematical expression for the reaction quotient, $Q_c$, for each of the following reactions:
(a) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
(b) $4NH_3(g) + 5O_2(g) \rightleftharpoons 4NO(g) + 6H_2O(g)$
(c) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$
(d) $CO_2(g) + H_2(g) \rightleftharpoons CO(g) + H_2O(g)$
(e) $NH_4Cl(s) \rightleftharpoons NH_3(g) + HCl(g)$
(f) $2Pb(NO_3)_2(s) \rightleftharpoons 2PbO(s) + 4NO_2(g) + O_2(g)$
(g) $2H_2(g) + O_2(g) \rightleftharpoons 2H_2O(l)$
(h) $S_8(g) \rightleftharpoons 8S(g)$

17. The initial concentrations or pressures of reactants and products are given for each of the following systems. Calculate the reaction quotient and determine the direction in which each system will proceed to reach equilibrium.
(a) $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$ \[ K_c = 17; \] \[ [NH_3] = 0.20 \text{ M}, [N_2] = 1.00 \text{ M}, [H_2] = 1.00 \text{ M} \]
(b) $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$ \[ K_P = 6.8 \times 10^4; \text{ initial pressures: } NH_3 = 3.0 \text{ atm}, N_2 = 2.0 \text{ atm}, H_2 = 1.0 \text{ atm} \]
(c) $2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$ \[ K_c = 0.230; \] \[ [SO_3] = 0.00 \text{ M}, [SO_2] = 1.00 \text{ M}, [O_2] = 1.00 \text{ M} \]
(d) $2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$ \[ K_P = 16.5; \text{ initial pressures: } SO_3 = 1.00 \text{ atm}, SO_2 = 1.00 \text{ atm}, O_2 = 1.00 \text{ atm} \]
(e) $2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl(g)$ \[ K_c = 4.6 \times 10^4; \] \[ [NO] = 1.00 \text{ M}, [Cl_2] = 1.00 \text{ M}, [NOCl] = 0 \text{ M} \]
(f) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$ \[ K_P = 0.050; \text{ initial pressures: } NO = 10.0 \text{ atm}, N_2 = O_2 = 5 \text{ atm} \]

18. The initial concentrations or pressures of reactants and products are given for each of the following systems. Calculate the reaction quotient and determine the direction in which each system will proceed to reach equilibrium.
(a) $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$ \[ K_c = 17; \] \[ [NH_3] = 0.50 \text{ M}, [N_2] = 0.15 \text{ M}, [H_2] = 0.12 \text{ M} \]
(b) $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$ \[ K_P = 6.8 \times 10^4; \text{ initial pressures: } NH_3 = 2.00 \text{ atm}, N_2 = 10.00 \text{ atm}, H_2 = 10.00 \text{ atm} \]
(c) $2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$ \[ K_c = 0.230; \] \[ [SO_3] = 2.00 \text{ M}, [SO_2] = 2.00 \text{ M}, [O_2] = 2.00 \text{ M} \]
(d) $2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$ \[ K_P = 6.5 \text{ atm}; \text{ initial pressures: } SO_3 = 1.00 \text{ atm}, O_2 = 1.130 \text{ atm}, SO_3 = 0 \text{ atm} \]
(e) $2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl(g)$ \[ K_P = 2.5 \times 10^3; \text{ initial pressures: } NO = 1.00 \text{ atm}, Cl_2 = 1.00 \text{ atm}, NOCl = 0 \text{ atm} \]
(f) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$ \[ K_c = 0.050; \] \[ [N_2] = 0.100 \text{ M}, [O_2] = 0.200 \text{ M}, [NO] = 1.00 \text{ M} \]

19. The following reaction has $K_P = 4.50 \times 10^{-5}$ at 720 K.
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

If a reaction vessel is filled with each gas to the partial pressures listed, in which direction will it shift to reach equilibrium? $P(NH_3) = 93 \text{ atm}$, $P(N_2) = 48 \text{ atm}$, and $P(H_2) = 52$.

20. Determine if the following system is at equilibrium. If not, in which direction will the system need to shift to reach equilibrium?
$SO_2Cl_2(g) \rightleftharpoons SO_2(g) + Cl_2(g)$

$[SO_2Cl_2] = 0.12 \text{ M}, [Cl_2] = 0.16 \text{ M}$ and $[SO_2] = 0.050 \text{ M}$. $K_c$ for the reaction is 0.078.
21. Which of the systems described in Exercise 13.15 give homogeneous equilibria? Which give heterogeneous equilibria?

22. Which of the systems described in Exercise 13.16 give homogeneous equilibria? Which give heterogeneous equilibria?

23. For which of the reactions in Exercise 13.15 does $K_c$ (calculated using concentrations) equal $K_P$ (calculated using pressures)?

24. For which of the reactions in Exercise 13.16 does $K_c$ (calculated using concentrations) equal $K_P$ (calculated using pressures)?

25. Convert the values of $K_c$ to values of $K_P$ or the values of $K_P$ to values of $K_c$.

   (a) $N_2(g) + 3H_2(g) ⇌ 2NH_3(g) \quad K_c = 0.50$ at 400 °C

   (b) $H_2(g) + I_2(g) ⇌ 2HI(g) \quad K_c = 50.2$ at 448 °C

   (c) $Na_2SO_4 \cdot 10H_2O(s) ⇌ Na_2SO_4(s) + 10H_2O(g) \quad K_P = 4.08 \times 10^{-25}$ at 25 °C

   (d) $H_2O(l) ⇌ H_2O(g) \quad K_P = 0.122$ at 50 °C

26. Convert the values of $K_c$ to values of $K_P$ or the values of $K_P$ to values of $K_c$.

   (a) $Cl_2(g) + Br_2(g) ⇌ 2BrCl(g) \quad K_c = 4.7 \times 10^{-2}$ at 25 °C

   (b) $2SO_2(g) + O_2(g) ⇌ 2SO_3(g) \quad K_P = 48.2$ at 500 °C

   (c) $CaCl_2 \cdot 6H_2O(s) ⇌ CaCl_2(s) + 6H_2O(g) \quad K_P = 5.09 \times 10^{-44}$ at 25 °C

   (d) $H_2O(l) ⇌ H_2O(g) \quad K_P = 0.196$ at 60 °C

27. What is the value of the equilibrium constant expression for the change $H_2O(l) ⇌ H_2O(g)$ at 30 °C?

28. Write the expression of the reaction quotient for the ionization of HOCN in water.

29. Write the reaction quotient expression for the ionization of NH$_3$ in water.

30. What is the approximate value of the equilibrium constant $K_P$ for the change $C_2H_4OC_2H_4(l) ⇌ C_3H_5OC_2H_5(g)$ at 25 °C. (Vapor pressure was described in the previous chapter on liquids and solids; refer back to this chapter to find the relevant information needed to solve this problem.)

13.3 Shifting Equilibria: Le Châtelier’s Principle

31. The following equation represents a reversible decomposition:

   $CaCO_3(s) ⇌ CaO(s) + CO_2(g)$

Under what conditions will decomposition in a closed container proceed to completion so that no CaCO$_3$ remains?

32. Explain how to recognize the conditions under which changes in pressure would affect systems at equilibrium.

33. What property of a reaction can we use to predict the effect of a change in temperature on the value of an equilibrium constant?

34. What would happen to the color of the solution in part (b) of Figure 13.8 if a small amount of NaOH were added and Fe(OH)$_3$ precipitated? Explain your answer.

35. The following reaction occurs when a burner on a gas stove is lit:

   $CH_4(g) + 2O_2(g) ⇌ CO_2(g) + 2H_2O(g)$

Is an equilibrium among CH$_4$, O$_2$, CO$_2$, and H$_2$O established under these conditions? Explain your answer.
36. A necessary step in the manufacture of sulfuric acid is the formation of sulfur trioxide, \( \text{SO}_3 \), from sulfur dioxide, \( \text{SO}_2 \), and oxygen, \( \text{O}_2 \), shown here. At high temperatures, the rate of formation of \( \text{SO}_3 \) is higher, but the equilibrium amount (concentration or partial pressure) of \( \text{SO}_3 \) is lower than it would be at lower temperatures.

\[
2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)
\]

(a) Does the equilibrium constant for the reaction increase, decrease, or remain about the same as the temperature increases?

(b) Is the reaction endothermic or exothermic?

37. Suggest four ways in which the concentration of hydrazine, \( \text{N}_2\text{H}_4 \), could be increased in an equilibrium described by the following equation:

\[
\text{N}_2(g) + 2\text{H}_2(g) \rightleftharpoons \text{N}_2\text{H}_4(g) \quad \Delta H = 95 \text{ kJ}
\]

38. Suggest four ways in which the concentration of \( \text{PH}_3 \) could be increased in an equilibrium described by the following equation:

\[
\text{P}_4(g) + 6\text{H}_2(g) \rightleftharpoons 4\text{PH}_3(g) \quad \Delta H = 110.5 \text{ kJ}
\]

39. How will an increase in temperature affect each of the following equilibria? How will a decrease in the volume of the reaction vessel affect each?

(a) \( 2\text{NH}_3(g) \rightleftharpoons \text{N}_2(g) + 3\text{H}_2(g) \quad \Delta H = 92 \text{ kJ} \)

(b) \( \text{N}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}(g) \quad \Delta H = 181 \text{ kJ} \)

(c) \( 2\text{O}_3(g) \rightleftharpoons 3\text{O}_2(g) \quad \Delta H = -285 \text{ kJ} \)

(d) \( \text{CaO(s)} + \text{CO}_2(g) \rightleftharpoons \text{CaCO}_3(s) \quad \Delta H = -176 \text{ kJ} \)

40. How will an increase in temperature affect each of the following equilibria? How will a decrease in the volume of the reaction vessel affect each?

(a) \( 2\text{H}_2\text{O}(g) \rightleftharpoons 2\text{H}_2(g) + \text{O}_2(g) \quad \Delta H = 484 \text{ kJ} \)

(b) \( \text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g) \quad \Delta H = -92.2 \text{ kJ} \)

(c) \( 2\text{Br}(g) \rightleftharpoons \text{Br}_2(g) \quad \Delta H = -224 \text{ kJ} \)

(d) \( \text{H}_2(g) + \text{I}_2(s) \rightleftharpoons 2\text{HI}(g) \quad \Delta H = 53 \text{ kJ} \)

41. Water gas is a 1:1 mixture of carbon monoxide and hydrogen gas and is called water gas because it is formed from steam and hot carbon in the following reaction: \( \text{H}_2\text{O}(g) + \text{C}(s) \rightleftharpoons \text{H}_2(g) + \text{CO}(g) \). Methanol, a liquid fuel that could possibly replace gasoline, can be prepared from water gas and hydrogen at high temperature and pressure in the presence of a suitable catalyst.

(a) Write the expression for the equilibrium constant \( K_c \) for the reversible reaction

\[
2\text{H}_2(g) + \text{CO}(g) \rightleftharpoons \text{CH}_3\text{OH}(g) \quad \Delta H = -90.2 \text{ kJ}
\]

(b) What will happen to the concentrations of \( \text{H}_2 \), \( \text{CO} \), and \( \text{CH}_3\text{OH} \) at equilibrium if more \( \text{H}_2 \) is added?

(c) What will happen to the concentrations of \( \text{H}_2 \), \( \text{CO} \), and \( \text{CH}_3\text{OH} \) at equilibrium if \( \text{CO} \) is removed?

(d) What will happen to the concentrations of \( \text{H}_2 \), \( \text{CO} \), and \( \text{CH}_3\text{OH} \) at equilibrium if \( \text{CH}_3\text{OH} \) is added?

(e) What will happen to the concentrations of \( \text{H}_2 \), \( \text{CO} \), and \( \text{CH}_3\text{OH} \) at equilibrium if the temperature of the system is increased?

(f) What will happen to the concentrations of \( \text{H}_2 \), \( \text{CO} \), and \( \text{CH}_3\text{OH} \) at equilibrium if more catalyst is added?
42. Nitrogen and oxygen react at high temperatures.
(a) Write the expression for the equilibrium constant ($K_c$) for the reversible reaction

$$\text{N}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}(g) \quad \Delta H = 181 \text{ kJ}$$

(b) What will happen to the concentrations of $\text{N}_2$, $\text{O}_2$, and $\text{NO}$ at equilibrium if more $\text{O}_2$ is added?
(c) What will happen to the concentrations of $\text{N}_2$, $\text{O}_2$, and $\text{NO}$ at equilibrium if $\text{N}_2$ is removed?
(d) What will happen to the concentrations of $\text{N}_2$, $\text{O}_2$, and $\text{NO}$ at equilibrium if NO is added?
(e) What will happen to the concentrations of $\text{N}_2$, $\text{O}_2$, and $\text{NO}$ at equilibrium if the pressure on the system is increased by reducing the volume of the reaction vessel?
(f) What will happen to the concentrations of $\text{N}_2$, $\text{O}_2$, and $\text{NO}$ at equilibrium if the temperature of the system is increased?
(g) What will happen to the concentrations of $\text{N}_2$, $\text{O}_2$, and $\text{NO}$ at equilibrium if a catalyst is added?

43. Water gas, a mixture of $\text{H}_2$ and CO, is an important industrial fuel produced by the reaction of steam with red hot coke, essentially pure carbon.
(a) Write the expression for the equilibrium constant for the reversible reaction

$$\text{C}(s) + \text{H}_2\text{O}(g) \rightleftharpoons \text{CO}(g) + \text{H}_2(g) \quad \Delta H = 131.30 \text{ kJ}$$

(b) What will happen to the concentration of each reactant and product at equilibrium if more C is added?
(c) What will happen to the concentration of each reactant and product at equilibrium if $\text{H}_2\text{O}$ is removed?
(d) What will happen to the concentration of each reactant and product at equilibrium if CO is added?
(e) What will happen to the concentration of each reactant and product at equilibrium if the temperature of the system is increased?

44. Pure iron metal can be produced by the reduction of iron(III) oxide with hydrogen gas.
(a) Write the expression for the equilibrium constant ($K_c$) for the reversible reaction

$$\text{Fe}_2\text{O}_3(s) + 3\text{H}_2(g) \rightleftharpoons 2\text{Fe}(s) + 3\text{H}_2\text{O}(g) \quad \Delta H = 98.7 \text{ kJ}$$

(b) What will happen to the concentration of each reactant and product at equilibrium if more Fe is added?
(c) What will happen to the concentration of each reactant and product at equilibrium if $\text{H}_2\text{O}$ is removed?
(d) What will happen to the concentration of each reactant and product at equilibrium if $\text{H}_2$ is added?
(e) What will happen to the concentration of each reactant and product at equilibrium if the pressure on the system is increased by reducing the volume of the reaction vessel?
(f) What will happen to the concentration of each reactant and product at equilibrium if the temperature of the system is increased?

45. Ammonia is a weak base that reacts with water according to this equation:

$$\text{NH}_3(aq) + \text{H}_2\text{O}(l) \rightleftharpoons \text{NH}_4^+(aq) + \text{OH}^-(aq)$$

Will any of the following increase the percent of ammonia that is converted to the ammonium ion in water?
(a) Addition of NaOH
(b) Addition of HCl
(c) Addition of $\text{NH}_4\text{Cl}$
46. Acetic acid is a weak acid that reacts with water according to this equation:

\[
\text{CH}_3\text{CO}_2\text{H}^{(aq)} + \text{H}_2\text{O}^{(aq)} \rightleftharpoons \text{H}_3\text{O}^{+(aq)} + \text{CH}_3\text{CO}_2^{-}\text{(aq)}
\]

Will any of the following increase the percent of acetic acid that reacts and produces \(\text{CH}_3\text{CO}_2^{-}\) ion?

(a) Addition of HCl
(b) Addition of NaOH
(c) Addition of NaCH\(_3\)CO\(_2\)

47. Suggest two ways in which the equilibrium concentration of Ag\(^+\) can be reduced in a solution of Na\(^+\), Cl\(^-\), Ag\(^+\), and NO\(_3\)^\(-\), in contact with solid AgCl.

\[
\text{Na}^+(aq) + \text{Cl}^-(aq) + \text{Ag}^+(aq) + \text{NO}_3^{-}(aq) \rightleftharpoons \text{AgCl(s)} + \text{Na}^+(aq) + \text{NO}_3^{-}(aq)
\]

\(\Delta H = -65.9\) kJ

48. How can the pressure of water vapor be increased in the following equilibrium?

\[
\text{H}_2\text{O}(l) = \text{H}_2\text{O}(g) \quad \Delta H = 41\) kJ

49. Additional solid silver sulfate, a slightly soluble solid, is added to a solution of silver ion and sulfate ion at equilibrium with solid silver sulfate.

\[
2\text{Ag}^+(aq) + \text{SO}_4^{2-}(aq) \rightleftharpoons \text{Ag}_2\text{SO}_4(s)
\]

Which of the following will occur?

(a) Ag\(^+\) or SO\(_4^{2-}\) concentrations will not change.
(b) The added silver sulfate will dissolve.
(c) Additional silver sulfate will form and precipitate from solution as Ag\(^+\) ions and SO\(_4^{2-}\) ions combine.
(d) The Ag\(^+\) ion concentration will increase and the SO\(_4^{2-}\) ion concentration will decrease.

50. The amino acid alanine has two isomers, α-alanine and β-alanine. When equal masses of these two compounds are dissolved in equal amounts of a solvent, the solution of α-alanine freezes at the lowest temperature. Which form, α-alanine or β-alanine, has the larger equilibrium constant for ionization (HX \(\rightleftharpoons\) H\(^+\) + X\(^-\))?

13.4 Equilibrium Calculations

51. A reaction is represented by this equation: \(A(aq) + 2B(aq) \rightleftharpoons 2C(aq)\) \(K_c = 1 \times 10^3\)

(a) Write the mathematical expression for the equilibrium constant.

(b) Using concentrations \(\leq 1\) M, make up two sets of concentrations that describe a mixture of A, B, and C at equilibrium.

52. A reaction is represented by this equation: \(2W(aq) \rightleftharpoons X(aq) + 2Y(aq)\) \(K_c = 5 \times 10^{-4}\)

(a) Write the mathematical expression for the equilibrium constant.

(b) Using concentrations \(\leq 1\) M, make up two sets of concentrations that describe a mixture of W, X, and Y at equilibrium.

53. What is the value of the equilibrium constant at 500 °C for the formation of NH\(_3\) according to the following equation?

\[
\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)
\]

An equilibrium mixture of NH\(_3\)(g), H\(_2\)(g), and N\(_2\)(g) at 500 °C was found to contain 1.35 M H\(_2\), 1.15 M N\(_2\), and 4.12 \(\times 10^{-1}\) M NH\(_3\).
54. Hydrogen is prepared commercially by the reaction of methane and water vapor at elevated temperatures.

\[
\text{CH}_4(g) + \text{H}_2\text{O}(g) \rightleftharpoons 3\text{H}_2(g) + \text{CO}(g)
\]

What is the equilibrium constant for the reaction if a mixture at equilibrium contains gases with the following concentrations: \( \text{CH}_4, 0.126 \ \text{M} \); \( \text{H}_2\text{O}, 0.242 \ \text{M} \); \( \text{CO}, 0.126 \ \text{M} \); \( \text{H}_2, 1.15 \ \text{M} \), at a temperature of 760 °C?

55. A 0.72-mol sample of \( \text{PCl}_5 \) is put into a 1.00-L vessel and heated. At equilibrium, the vessel contains 0.40 mol of \( \text{PCl}_3(g) \) and 0.40 mol of \( \text{Cl}_2(g) \). Calculate the value of the equilibrium constant for the decomposition of \( \text{PCl}_5 \) to \( \text{PCl}_3 \) and \( \text{Cl}_2 \) at this temperature.

56. At 1 atm and 25 °C, \( \text{NO}_2 \) with an initial concentration of 1.00 M is 3.3 \( \times \) 10\(^{-3} \)\% decomposed into \( \text{NO} \) and \( \text{O}_2 \). Calculate the value of the equilibrium constant for the reaction.

\[
2\text{NO}_2(g) \rightleftharpoons 2\text{NO}(g) + \text{O}_2(g)
\]

57. Calculate the value of the equilibrium constant \( K_P \) for the reaction \( 2\text{NO}(g) + \text{Cl}_2(g) \rightleftharpoons 2\text{NOCl}(g) \) from these equilibrium pressures: \( \text{NO}, 0.050 \ \text{atm} \); \( \text{Cl}_2, 0.30 \ \text{atm} \); \( \text{NOCl}, 1.2 \ \text{atm} \).

58. When heated, iodine vapor dissociates according to this equation:

\[
\text{I}_2(g) \rightleftharpoons 2\text{I}(g)
\]

At 1274 K, a sample exhibits a partial pressure of \( \text{I}_2 \) of 0.1122 atm and a partial pressure due to \( \text{I} \) atoms of 0.1378 atm. Determine the value of the equilibrium constant, \( K_P \), for the decomposition at 1274 K.

59. A sample of ammonium chloride was heated in a closed container.

\[
\text{NH}_4\text{Cl}(s) = \text{NH}_3(g) + \text{HCl}(g)
\]

At equilibrium, the pressure of \( \text{NH}_3(g) \) was found to be 1.75 atm. What is the value of the equilibrium constant \( K_P \) for the decomposition at this temperature?

60. At a temperature of 60 °C, the vapor pressure of water is 0.196 atm. What is the value of the equilibrium constant \( K_P \) for the transformation at 60 °C?

\[
\text{H}_2\text{O}(l) = \text{H}_2\text{O}(g)
\]
61. Complete the changes in concentrations (or pressure, if requested) for each of the following reactions.

(a) \[ 2\text{SO}_3(g) \rightleftharpoons 2\text{SO}_2(g) + \text{O}_2(g) \]

\[ \_\_\_\_ \_\_ + x \]

\[ \_\_\_\_ \_\_ 0.125 \text{ M} \]

(b) \[ 4\text{NH}_3(g) + 3\text{O}_2(g) \rightleftharpoons 2\text{N}_2(g) + 6\text{H}_2\text{O}(g) \]

\[ \_\_\_\_ 3x \_\_\_\_ \_\_ \]

\[ \_\_\_\_ \_\_ 0.24 \text{ M} \]

(c) Change in pressure:

\[ 2\text{CH}_4(g) \rightleftharpoons \text{C}_2\text{H}_2(g) + 3\text{H}_2(g) \]

\[ \_\_\_\_ x \_\_\_\_ \_\_ \]

\[ \_\_\_\_ 25 \text{ torr} \]

(d) Change in pressure:

\[ \text{CH}_4(g) + \text{H}_2\text{O}(g) \rightleftharpoons \text{CO}(g) + 3\text{H}_2(g) \]

\[ \_\_\_\_ x \_\_\_\_ \_\_ \]

\[ \_\_\_\_ 5 \text{ atm} \]

(e) \[ \text{NH}_4\text{Cl}(s) \rightleftharpoons \text{NH}_3(g) + \text{HCl}(g) \]

\[ x \_\_\_\_ \_\_ \]

\[ 1.03 \times 10^{-4} \text{ M} \]

(f) Change in pressure:

\[ \text{Ni}(s) + 4\text{CO}(g) \rightleftharpoons \text{Ni(CO)}_4(g) \]

\[ 4x \_\_\_\_ \_\_ \]

\[ 0.40 \text{ atm} \]
62. Complete the changes in concentrations (or pressure, if requested) for each of the following reactions.

(a) \[ 2\text{H}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{H}_2\text{O}(g) \]
\[ +2x \]
\[ 1.50 \text{M} \]

(b) \[ \text{CS}_2(g) + 4\text{H}_2(g) \rightleftharpoons \text{CH}_4(g) + 2\text{H}_2\text{S}(g) \]
\[ x \]
\[ 0.020 \text{M} \]

(c) Change in pressure:
\[ \text{H}_2(g) + \text{Cl}_2(g) \rightleftharpoons 2\text{HCl}(g) \]
\[ x \]
\[ 1.50 \text{atm} \]

(d) Change in pressure:
\[ 2\text{NH}_3(g) + 2\text{O}_2(g) \rightleftharpoons \text{N}_2\text{O}(g) + 3\text{H}_2\text{O}(g) \]
\[ x \]
\[ 60.6 \text{torr} \]

(e) \[ \text{NH}_4\text{HS}(s) \rightleftharpoons \text{NH}_3(g) + \text{H}_2\text{S}(g) \]
\[ x \]
\[ 9.8 \times 10^{-6} \text{M} \]

(f) Change in pressure:
\[ \text{Fe}(s) + 5\text{CO}(g) \rightleftharpoons \text{Fe}(\text{CO})_5(g) \]
\[ x \]
\[ 0.012 \text{atm} \]

63. Why are there no changes specified for Ni in Exercise 13.61, part (f)? What property of Ni does change?

64. Why are there no changes specified for NH$_4$HS in Exercise 13.62, part (e)? What property of NH$_4$HS does change?

65. Analysis of the gases in a sealed reaction vessel containing NH$_3$, N$_2$, and H$_2$ at equilibrium at 400 °C established the concentration of N$_2$ to be 1.2 M and the concentration of H$_2$ to be 0.24 M.

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g) \]
\[ K_c = 0.50 \text{ at } 400 \degree \text{C} \]

Calculate the equilibrium molar concentration of NH$_3$.

66. Calculate the number of moles of HI that are at equilibrium with 1.25 mol of H$_2$ and 1.25 mol of I$_2$ in a 5.00−L flask at 448 °C.

\[ \text{H}_2 + \text{I}_2 \rightleftharpoons 2\text{HI} \]
\[ K_c = 50.2 \text{ at } 448 \degree \text{C} \]

67. What is the pressure of BrCl in an equilibrium mixture of Cl$_2$, Br$_2$, and BrCl if the pressure of Cl$_2$ in the mixture is 0.115 atm and the pressure of Br$_2$ in the mixture is 0.450 atm?

\[ \text{Cl}_2(g) + \text{Br}_2(g) \rightleftharpoons 2\text{BrCl}(g) \]
\[ K_P = 4.7 \times 10^{-2} \]

68. What is the pressure of CO$_2$ in a mixture at equilibrium that contains 0.50 atm H$_2$, 2.0 atm of H$_2$O, and 1.0 atm of CO at 990 °C?

\[ \text{H}_2(g) + \text{CO}_2(g) \rightleftharpoons \text{H}_2\text{O}(g) + \text{CO}(g) \]
\[ K_P = 1.6 \text{ at } 990 \degree \text{C} \]
69. Cobalt metal can be prepared by reducing cobalt(II) oxide with carbon monoxide.

\[
\text{CoO(s) + CO(g) } \rightleftharpoons \text{ Co(s) + CO}_2(g) \quad K_c = 4.90 \times 10^2 \text{ at } 550 \, ^\circ\text{C}
\]

What concentration of CO remains in an equilibrium mixture with [CO\textsubscript{2}] = 0.100 M?

70. Carbon reacts with water vapor at elevated temperatures.

\[
\text{C(s) + H}_2\text{O(g) } \rightleftharpoons \text{ CO(g) + H}_2\text{(g)} \quad K_c = 0.2 \text{ at } 1000 \, ^\circ\text{C}
\]

What is the concentration of CO in an equilibrium mixture with [H\textsubscript{2}O] = 0.500 M at 1000 °C?

71. Sodium sulfate 10−hydrate, Na\textsubscript{2}SO\textsubscript{4}·10H\textsubscript{2}O, dehydrates according to the equation

\[
\text{Na}_2\text{SO}_4\cdot10\text{H}_2\text{O(s) } \rightleftharpoons \text{ Na}_2\text{SO}_4(s) + 10\text{H}_2\text{O(g)} \quad K_P = 4.08 \times 10^{-25} \text{ at } 25 \, ^\circ\text{C}
\]

What is the pressure of water vapor at equilibrium with a mixture of Na\textsubscript{2}SO\textsubscript{4}·10H\textsubscript{2}O and NaSO\textsubscript{4}?

72. Calcium chloride 6−hydrate, CaCl\textsubscript{2}·6H\textsubscript{2}O, dehydrates according to the equation

\[
\text{CaCl}_2\cdot6\text{H}_2\text{O(s) } \rightleftharpoons \text{ CaCl}_2(s) + 6\text{H}_2\text{O(g)} \quad K_P = 5.09 \times 10^{-44} \text{ at } 25 \, ^\circ\text{C}
\]

What is the pressure of water vapor at equilibrium with a mixture of CaCl\textsubscript{2}·6H\textsubscript{2}O and CaCl\textsubscript{2}?

73. A student solved the following problem and found the equilibrium concentrations to be [SO\textsubscript{2}] = 0.590 M, [O\textsubscript{2}] = 0.0450 M, and [SO\textsubscript{3}] = 0.260 M. How could this student check the work without reworking the problem? The problem was: For the following reaction at 600 °C:

\[
2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g) \quad K_c = 4.32
\]

What are the equilibrium concentrations of all species in a mixture that was prepared with [SO\textsubscript{3}] = 0.500 M, [SO\textsubscript{2}] = 0 M, and [O\textsubscript{2}] = 0.350 M?

74. A student solved the following problem and found [N\textsubscript{2}O\textsubscript{4}] = 0.16 M at equilibrium. How could this student recognize that the answer was wrong without reworking the problem? The problem was: What is the equilibrium concentration of N\textsubscript{2}O\textsubscript{4} in a mixture formed from a sample of NO\textsubscript{2} with a concentration of 0.10 M?

\[
2\text{NO}_2(g) \rightleftharpoons \text{N}_2\text{O}_4(g) \quad K_c = 160
\]

75. Assume that the change in concentration of N\textsubscript{2}O\textsubscript{4} is small enough to be neglected in the following problem.

(a) Calculate the equilibrium concentration of both species in 1.00 L of a solution prepared from 0.129 mol of N\textsubscript{2}O\textsubscript{4} with chloroform as the solvent.

\[
\text{N}_2\text{O}_4(g) \rightleftharpoons 2\text{NO}_2(g) \quad K_c = 1.07 \times 10^{-5} \text{ in chloroform}
\]

(b) Show that the change is small enough to be neglected.

76. Assume that the change in concentration of COCl\textsubscript{2} is small enough to be neglected in the following problem.

(a) Calculate the equilibrium concentration of all species in an equilibrium mixture that results from the decomposition of COCl\textsubscript{2} with an initial concentration of 0.3166 M.

\[
\text{COCl}_2(g) \rightleftharpoons \text{CO(g) + Cl}_2(g) \quad K_c = 2.2 \times 10^{-10}
\]

(b) Show that the change is small enough to be neglected.

77. Assume that the change in pressure of H\textsubscript{2}S is small enough to be neglected in the following problem.

(a) Calculate the equilibrium pressures of all species in an equilibrium mixture that results from the decomposition of H\textsubscript{2}S with an initial pressure of 0.824 atm.

\[
2\text{H}_2\text{S}(g) \rightleftharpoons 2\text{H}_2(g) + \text{S}_2(g) \quad K_P = 2.2 \times 10^{-6}
\]

(b) Show that the change is small enough to be neglected.
78. What are all concentrations after a mixture that contains \([H_2O] = 1.00 \text{ M}\) and \([Cl_2O] = 1.00 \text{ M}\) comes to equilibrium at 25 °C?

\[H_2O(g) + Cl_2O(g) \rightleftharpoons 2HOCI(g) \quad K_c = 0.0900\]

79. What are the concentrations of PCl₅, PCl₃, and Cl₂ in an equilibrium mixture produced by the decomposition of a sample of pure PCl₅ with \([PCl_5] = 2.00 \text{ M}\)?

\[PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g) \quad K_c = 0.0211\]

80. Calculate the pressures of all species at equilibrium in a mixture of NOCl, NO, and Cl₂ produced when a sample of NOCl with a pressure of 10.0 atm comes to equilibrium according to this reaction:

\[2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g) \quad K_P = 4.0 \times 10^{-4}\]

81. Calculate the equilibrium concentrations that result when 0.25 M O₂ and 1.0 M HCl react and come to equilibrium.

\[4HCl(g) + O_2(g) \rightleftharpoons 2Cl_2(g) + 2H_2O(g) \quad K_c = 3.1 \times 10^{13}\]

82. Calculate the pressures of NO, Cl₂, and NOCl in an equilibrium mixture produced by the reaction of a starting mixture with 4.0 atm NO and 2.0 atm Cl₂. (Hint: \(K_P\) is small; assume the reverse reaction goes to completion then comes back to equilibrium.)

\[2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl(g) \quad K_P = 2.5 \times 10^3\]

83. One of the important reactions in the formation of smog is represented by the equation

\[O_3(g) + NO(g) \rightleftharpoons NO_2(g) + O_2(g) \quad K_P = 6.0 \times 10^{34}\]

What is the pressure of O₃ remaining after a mixture of O₃ with a pressure of \(1.2 \times 10^{-8}\) atm and NO with a pressure of \(1.2 \times 10^{-8}\) atm comes to equilibrium? (Hint: \(K_P\) is large; assume the reaction goes to completion then comes back to equilibrium.)

84. Calculate the pressures of NO, Cl₂, and NOCl in an equilibrium mixture produced by the reaction of a starting mixture with 4.0 atm NO and 2.0 atm Cl₂. (Hint: \(K_P\) is small; assume the reverse reaction goes to completion then comes back to equilibrium.)

\[2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl(g) \quad K_P = 2.5 \times 10^3\]

85. Calculate the number of grams of HI that are at equilibrium with 1.25 mol of H₂ and 63.5 g of iodine at 448 °C.

\[H_2 + I_2 \rightleftharpoons 2HI \quad K_c = 50.2 \text{ at } 448 \text{ °C}\]

86. Butane exists as two isomers, \(n\)-butane and isobutane.

\[CH_3 - CH_2 - CH_2 - CH_3 \quad \rightleftharpoons \quad CH_3 - CH(CH_3)_2 \quad K_P = 2.5 \text{ at } 25 \text{ °C}\]

What is the pressure of isobutane in a container of the two isomers at equilibrium with a total pressure of 1.22 atm?

87. What is the minimum mass of CaCO₃ required to establish equilibrium at a certain temperature in a 6.50-L container if the equilibrium constant \((K_c)\) is 0.050 for the decomposition reaction of CaCO₃ at that temperature?

\[CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)\]
88. The equilibrium constant \( (K_c) \) for this reaction is 1.60 at 990 °C:

\[
\text{H}_2(\text{g}) + \text{CO}_2(\text{g}) \rightleftharpoons \text{H}_2\text{O}(\text{g}) + \text{CO}(\text{g})
\]

Calculate the number of moles of each component in the final equilibrium mixture obtained from adding 1.00 mol of \( \text{H}_2 \), 2.00 mol of \( \text{CO}_2 \), 0.750 mol of \( \text{H}_2\text{O} \), and 1.00 mol of \( \text{CO} \) to a 5.00-L container at 990 °C.

89. At 25 °C and at 1 atm, the partial pressures in an equilibrium mixture of \( \text{N}_2\text{O}_4 \) and \( \text{NO}_2 \) are \( P_{\text{N}_2\text{O}_4} = 0.70 \) atm and \( P_{\text{NO}_2} = 0.30 \) atm.

(a) Predict how the pressures of \( \text{NO}_2 \) and \( \text{N}_2\text{O}_4 \) will change if the total pressure increases to 9.0 atm. Will they increase, decrease, or remain the same?

(b) Calculate the partial pressures of \( \text{NO}_2 \) and \( \text{N}_2\text{O}_4 \) when they are at equilibrium at 9.0 atm and 25 °C.

90. In a 3.0-L vessel, the following equilibrium partial pressures are measured: \( \text{N}_2 \), 190 torr; \( \text{H}_2 \), 317 torr; \( \text{NH}_3 \), \( 1.00 \times 10^3 \) torr.

\[
\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightleftharpoons 2\text{NH}_3(\text{g})
\]

(a) How will the partial pressures of \( \text{H}_2 \), \( \text{N}_2 \), and \( \text{NH}_3 \) change if \( \text{H}_2 \) is removed from the system? Will they increase, decrease, or remain the same?

(b) Hydrogen is removed from the vessel until the partial pressure of nitrogen, at equilibrium, is 250 torr. Calculate the partial pressures of the other substances under the new conditions.

91. The equilibrium constant \( (K_c) \) for this reaction is 5.0 at a given temperature.

\[
\text{CO}(\text{g}) + \text{H}_2\text{O}(\text{g}) \rightleftharpoons \text{CO}_2(\text{g}) + \text{H}_2(\text{g})
\]

(a) On analysis, an equilibrium mixture of the substances present at the given temperature was found to contain 0.20 mol of \( \text{CO} \), 0.30 mol of water vapor, and 0.90 mol of \( \text{H}_2 \) in a liter. How many moles of \( \text{CO}_2 \) were there in the equilibrium mixture?

(b) Maintaining the same temperature, additional \( \text{H}_2 \) was added to the system, and some water vapor was removed by drying. A new equilibrium mixture was thereby established containing 0.40 mol of \( \text{CO} \), 0.30 mol of water vapor, and 1.2 mol of \( \text{H}_2 \) in a liter. How many moles of \( \text{CO}_2 \) were in the new equilibrium mixture? Compare this with the quantity in part (a), and discuss whether the second value is reasonable. Explain how it is possible for the water vapor concentration to be the same in the two equilibrium solutions even though some vapor was removed before the second equilibrium was established.

92. Antimony pentachloride decomposes according to this equation:

\[
\text{SbCl}_5(g) \rightleftharpoons \text{SbCl}_3(g) + \text{Cl}_2(g)
\]

An equilibrium mixture in a 5.00-L flask at 448 °C contains 3.85 g of \( \text{SbCl}_5 \), 9.14 g of \( \text{SbCl}_3 \), and 2.84 g of \( \text{Cl}_2 \). How many grams of each will be found if the mixture is transferred into a 2.00-L flask at the same temperature?

93. Consider the reaction between \( \text{H}_2 \) and \( \text{O}_2 \) at 1000 K

\[
2\text{H}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{H}_2\text{O}(g)
\]

\[K_P = \frac{(P_{\text{H}_2\text{O}})^2}{(P_{\text{O}_2})(P_{\text{H}_2})^2} = 1.33 \times 10^{20}\]

If 0.500 atm of \( \text{H}_2 \) and 0.500 atm of \( \text{O}_2 \) are allowed to come to equilibrium at this temperature, what are the partial pressures of the components?
94. An equilibrium is established according to the following equation
\[ \text{Hg}_2^{2+}(aq) + \text{NO}_3^{-}(aq) + 3\text{H}^+(aq) \rightleftharpoons 2\text{Hg}^{2+}(aq) + \text{HNO}_2(aq) + \text{H}_2\text{O}(l) \quad K_C = 4.6 \]

What will happen in a solution that is 0.20 M each in \( \text{Hg}_2^{2+} \), \( \text{NO}_3^{-} \), \( \text{H}^+ \), \( \text{Hg}^{2+} \), and \( \text{HNO}_2 \)?

(a) \( \text{Hg}_2^{2+} \) will be oxidized and \( \text{NO}_3^{-} \) reduced.

(b) \( \text{Hg}_2^{2+} \) will be reduced and \( \text{NO}_3^{-} \) oxidized.

(c) \( \text{Hg}^{2+} \) will be oxidized and \( \text{HNO}_2 \) reduced.

(d) \( \text{Hg}^{2+} \) will be reduced and \( \text{HNO}_2 \) oxidized.

(e) There will be no change because all reactants and products have an activity of 1.

95. Consider the equilibrium
\[ 4\text{NO}_2(g) + 6\text{H}_2\text{O}(g) \rightleftharpoons 4\text{NH}_3(g) + 7\text{O}_2(g) \]

(a) What is the expression for the equilibrium constant \( K_C \) of the reaction?

(b) How must the concentration of \( \text{NH}_3 \) change to reach equilibrium if the reaction quotient is less than the equilibrium constant?

(c) If the reaction were at equilibrium, how would a decrease in pressure (from an increase in the volume of the reaction vessel) affect the pressure of \( \text{NO}_2 \)?

(d) If the change in the pressure of \( \text{NO}_2 \) is 28 torr as a mixture of the four gases reaches equilibrium, how much will the pressure of \( \text{O}_2 \) change?

96. The binding of oxygen by hemoglobin (Hb), giving oxyhemoglobin (HbO\(_2\)), is partially regulated by the concentration of \( \text{H}_3\text{O}^+ \) and dissolved CO\(_2\) in the blood. Although the equilibrium is complicated, it can be summarized as
\[ \text{HbO}_2(aq) + \text{H}_3\text{O}^+(aq) + \text{CO}_2(g) \rightleftharpoons \text{CO}_2^-\text{Hb}^-\text{H}^+ + \text{O}_2(g) + \text{H}_2\text{O}(l) \]

(a) Write the equilibrium constant expression for this reaction.

(b) Explain why the production of lactic acid and CO\(_2\) in a muscle during exertion stimulates release of \( \text{O}_2 \) from the oxyhemoglobin in the blood passing through the muscle.

97. The hydrolysis of the sugar sucrose to the sugars glucose and fructose follows a first-order rate equation for the disappearance of sucrose.
\[ \text{C}_{12}\text{H}_{22}\text{O}_{11}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{C}_6\text{H}_{12}\text{O}_6(aq) + \text{C}_6\text{H}_{12}\text{O}_6(aq) \]

Rate = \( k[C_{12}H_{22}O_{11}] \)

In neutral solution, \( k = 2.1 \times 10^{-11}/s \) at 27 °C. (As indicated by the rate constant, this is a very slow reaction. In the human body, the rate of this reaction is sped up by a type of catalyst called an enzyme.) (Note: That is not a mistake in the equation—the products of the reaction, glucose and fructose, have the same molecular formulas, \( \text{C}_6\text{H}_{12}\text{O}_6 \), but differ in the arrangement of the atoms in their molecules). The equilibrium constant for the reaction is \( 1.36 \times 10^5 \) at 27 °C. What are the concentrations of glucose, fructose, and sucrose after a 0.150 M aqueous solution of sucrose has reached equilibrium? Remember that the activity of a solvent (the effective concentration) is 1.

98. The density of trifluoroacetic acid vapor was determined at 118.1 °C and 468.5 torr, and found to be 2.784 g/L. Calculate \( K_C \) for the association of the acid.
99. Liquid N₂O₃ is dark blue at low temperatures, but the color fades and becomes greenish at higher temperatures as the compound decomposes to NO and NO₂. At 25 °C, a value of $K_p = 1.91$ has been established for this decomposition. If 0.236 moles of N₂O₃ are placed in a 1.52-L vessel at 25 °C, calculate the equilibrium partial pressures of N₂O₃(g), NO₂(g), and NO(g).

100. A 1.00-L vessel at 400 °C contains the following equilibrium concentrations: N₂, 1.00 M; H₂, 0.50 M; and NH₃, 0.25 M. How many moles of hydrogen must be removed from the vessel to increase the concentration of nitrogen to 1.1 M?

101. A 0.010 M solution of the weak acid HA has an osmotic pressure (see chapter on solutions and colloids) of 0.293 atm at 25 °C. A 0.010 M solution of the weak acid HB has an osmotic pressure of 0.345 atm under the same conditions.

(a) Which acid has the larger equilibrium constant for ionization

HA [HA(aq) ⇌ A⁻(aq) + H⁺(aq)] or HB [HB(aq) ⇌ H⁺(aq) + B⁻(aq)]?

(b) What are the equilibrium constants for the ionization of these acids?

(Hint: Remember that each solution contains three dissolved species: the weak acid (HA or HB), the conjugate base (A⁻ or B⁻), and the hydrogen ion (H⁺). Remember that osmotic pressure (like all colligative properties) is related to the total number of solute particles. Specifically for osmotic pressure, those concentrations are described by molarities.)